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ABSTRACT

This concept module on time constants is one of thirteen modules that provide a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, postsecondary technical schools. Each of the thirteen concept modules discusses a single physics concept and how it is applied to each energy system. In this module, time constants are shown to be common to different energy systems and to serve, therefore, as a unifying concept. Included with the concept module are seven application modules that discuss a specific application of a single concept in a single energy system, and contain a student laboratory exercise. Laboratory modules included are Shock Isolation by Absorbers, Rate of Emptying Tanks, Resistance Capacitance Circuits, RL Circuits, Resistance Inductance Capacitance Circuits, Response Time of Thermocouples, and Time Constants of Heating and Cooling. (LRA)

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# UTC

UNIFIED TECHNICAL CONCEPTS

CONCEPT MODULE 12-0

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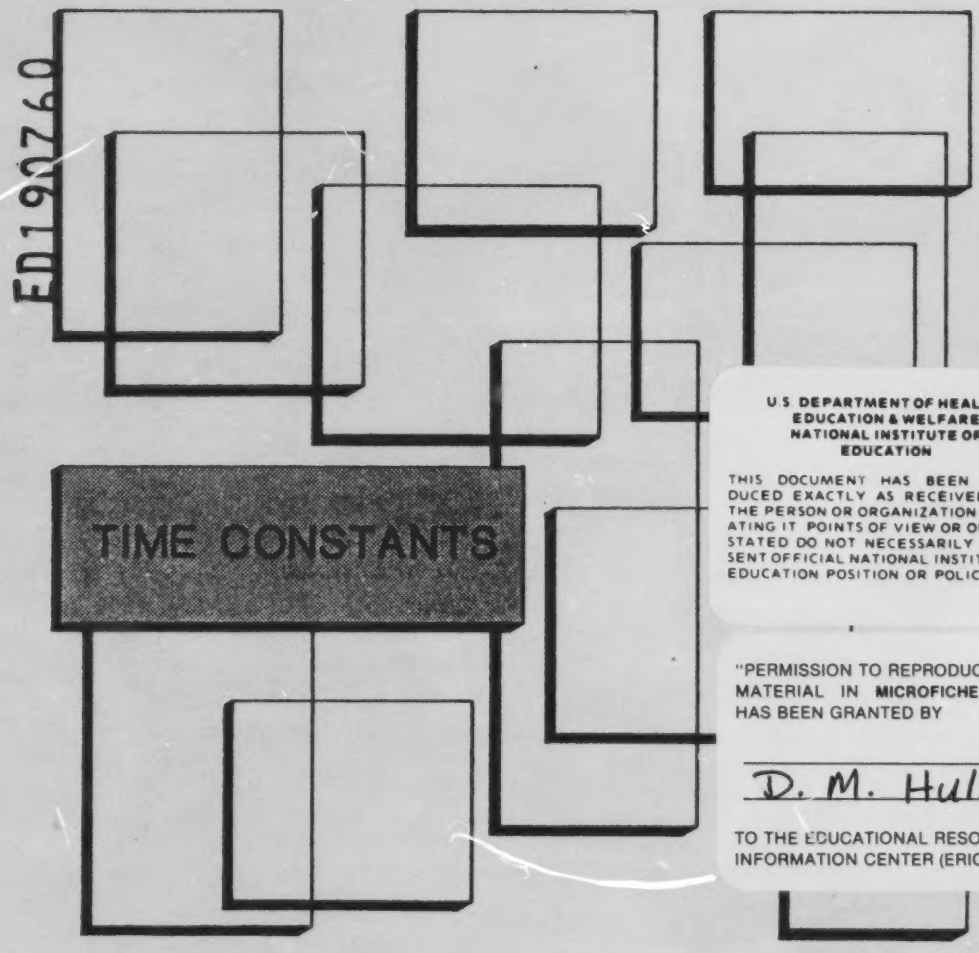


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## INTRODUCTION TO UTC

Complex, modern equipment often contains combinations of hydraulic, pneumatic, thermodynamic or optical subsystems, as well as mechanical and electrical components. These subsystems and components appear not in isolation but as interdependent parts of complex systems. Consequently, a modern technician must understand not only the separate subsystems but also their interrelationships and the roles they play in an integrated system.

Since physics provides the foundation for modern technology, physics instruction in technical programs cannot be overemphasized. Physics courses for technicians should achieve three major objectives:

- Provide students with all the basic physics principles that apply to their chosen field
- Reinforce these principles through the use of "real" examples and hands-on experience.
- Organize knowledge and skills gained in a manner that provides ease of application in solving systems-oriented problems.

These objectives must be accomplished in a minimum of instructional time, and in a manner flexible enough to meet the needs of a variety of technical specialties.

Unified Technical Concepts (UTC) is a flexible, laboratory-based physics instructional package designed to meet the specialized needs of students in two-year, post-secondary technical schools. UTC organizes technical physics into 13 basic concepts (Force, Work, etc.) that apply in four "energy systems" (Mechanical, Fluidal, Electrical, Thermal). Each of the 13 "concept" modules discusses a single physics concept and how it is applied to each energy system. Similarly, each "application" module discusses a specific application of a single concept in a single energy system, and contains a student laboratory exercise.

The recommended prerequisite for UTC is one year of high school algebra, but eight supplementary modules are available for students requiring additional skills or review in the areas of math, units and graphing.

UTC is an alternate method of presenting what heretofore has been a formidable and sometimes confusing subject for technicians. It provides laboratory experiments that apply to specific technological fields, which allows a student to relate more quickly to module content - to understand how it applies to a specific technology.

UTC also represents a unique approach to teaching applied physics. Its flexible modular format allows it to be tailored by physics and technology instructors to meet

specific needs of students in a specific technology.

## STRUCTURE OF UTC

The modern world is based on the control and application of energy. Technology is the means by which that energy is applied to accomplish a desired task. Technical physics, being primarily a study of the physical basis of the control and use of energy, can be categorized according to the type of energy involved. Thus, in UTC, physics is divided first into four "energy systems" - Mechanical, Fluidal, Electrical and Thermal.

The same divisions are characteristic of traditional approaches to teaching technical physics. A student usually begins by studying mechanics. Once this branch of physics (energy system) is mastered, electromagnetism, thermodynamics and hydraulics are studied (one energy system at a time).

UTC takes a different approach.

### CONCEPTS IN PHYSICS

The basic ideas, or "concepts," of physics appear in all energy systems, and these concepts relate to one another in similar ways in all systems. UTC unifies technical physics by presenting first the physics concept and then its application in each energy system.

The first concept studied, for example, is that of force, - the physical quantity that produces motion. The basic characteristics of force are followed by a discussion of how this concept applies to "forcelike quantities" in the following energy "systems":

- In mechanical systems force and torque cause motion of mass.
- In fluidal systems pressure difference causes motion of fluids.
- In electrical systems potential difference causes motion of mass.
- In thermal systems temperature difference causes motion of heat energy.

Problems, demonstrations and hands-on laboratory exercises are used to show the application of the concept in each energy system, and to specific devices or processes common in modern technology.

Thus, UTC provides a student with the basic concepts of physics reinforced by practical application and organized into a consistent whole; UTC is readily applied to future technical problems, whether in other courses or on the job.

## COMPONENTS OF UTC

UTC incorporates three types of modules.

### CONCEPT MODULES

Thirteen concept modules form the backbone of UTC (each presents a single concept in applied physics and discusses the application of that concept in each energy system):

1-0 Force.
2-0 Work.
3-0 Rate.
4-0 Momentum.
5-0 Resistance.
6-0 Power.
7-0 Potential and Kinetic Energy.
8-0 Force Transformers.
9-0 Energy Convertors.
10-0 Transducers.
11-0 Vibrations and Waves.
12-0 Time Constants.
13-0 Radiation.

### APPLICATION MODULES

Each of the 170 application modules discusses the application of a single physics concept in a single energy system, and each module includes a laboratory exercise in which students investigate the application. For example, 5E5 "Resistance of Wires" (located under Concept 5-0 "Resistance"), discusses the electrical resistance of wires. In the laboratory, students calculate and measure experimentally the resistance of various wires.

### SUPPLEMENTARY MODULES

In UTC, mathematics has been reduced to a minimum: one year of high school algebra is adequate preparation. The following eight supplementary modules are available for students requiring additional preparation or review:

S1 Formula Interpretation.
S2 International System of Units.
S3 Logarithms and Exponents.
S4 Angles and Triangles.
S5 Vectors and Scalars.
S6 Dimensional Analysis.
S7 Reading and Drawing Graphs.
S8 Precision, Accuracy, and Measurement.

## STRUCTURE OF A UTC MODULE

Each module in UTC contains seven basic elements:

**INTRODUCTION**

A brief description of the topics to be discussed, and their importance in technology, introduces each module.

**PREREQUISITES**

A list of prerequisite UTC modules, and notice of any special knowledge or skills required for successful completion of the module, are given.

**OBJECTIVES**

Specific tasks that a student should be able to accomplish after completing the module are stated in behavioral terms; objectives also are the basis of testing for UTC.

**DISCUSSION**

This segment is an explanation of the topic to be studied. It contains the appropriate physics as well as examples of its application. The discussion includes development of equations, example problems, figures and tables of useful data. It addresses each of the objectives (except those pertaining directly to a laboratory experiment).

**LABORATORY (called DEMONSTRATIONS in the Concept Modules)**

The laboratory section of UTC modules consists of the following three parts:

- "Materials." A list of all equipment and supplies required for the laboratory.
- "Procedure." A step-by-step set of instructions for performing the experiment.
- "Data." An organized format for recording all necessary data and observations.

**SELF CHECK**

A set of questions and problems based on the module objectives is included. It is intended for use by students to check their own understanding of module content.

**REFERENCES**

This list of sources directs students to additional information on topics discussed in the module.

TABLE OF CONTENTS


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| 12M1 | Shock Isolation by Absorbers |
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| 12E3 | RLC Circuits |
| 12T1 | Response Time of Thermocouples |
| 12T2 | Time Constants of Heating and Cooling |

## INTRODUCTION

In many practical situations, it is important to know how swiftly a system will give up or accept energy, will change its temperature, or will return to equilibrium after some initial displacement. For example, how much time is required for a heated piece of metal at some temperature T to cool to room temperature? How much time is required for a rotating record-player table to stop completely? How long must a battery be charged?

Some systems may not reach an equilibrium, or final state, until an extraordinary length of time has elapsed. As an example, a capacitor never becomes fully charged; however, in practice it may come so close to being charged in a given time that it can be considered "fully" charged.

Time constants — intervals of time during which a system undergoes a given change — are used to describe how fast systems are changing. If the time constant for a changing system is known, one can predict time required for the complete change to take place; for example, time required for a capacitor to reach 99% of a fully-charged condition. In this module, time constants will be shown to be common to different energy systems and to serve, therefore, as a unifying concept.

## PREREQUISITES

The student should have completed Concept Module 7-0, "Potential and Kinetic Energy," and should be able to use mathematical equations that include exponential terms.

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Steady state.
  Transient.
  Damping.

Time constant.
Half-life.
Decay constant.

- Draw and label a graph showing an exponentially-decaying function. Include on the graph the time constants $T_{1/2}$ and $T_{1/e} \equiv \tau$. Also, write an equation for the function shown on the graph.

- Given the initial temperature of a hot body, the ambient temperature of its surroundings, and the thermal time constant of the system, determine the temperature of the body after a specified time interval.

- Given the number of radioactive atoms in a sample and the decay constant, determine the number of atoms remaining after a specified time interval.

- Given the values of resistance, capacitance, and applied voltage in an RC electrical circuit, determine the time constant for the circuit, the time required for the capacitor voltage to reach 99% of the applied voltage, and the circuit current and capacitor voltage after a specified time interval.

- Explain how the concept of time constants can be applied to the following energy systems; give a specific example in each case:

  Mechanical translational.
  Mechanical rotational.
  Fluidal.
  Electrical.
  Thermal.
  Optical.
  Nuclear.

## DISCUSSION

Concept Module 3-0 discussed rates, all of which were constant and, therefore, did not vary with time. For example, if one gallon per minute was the given flow rate of a fluid through a pipe at any given time, flow rate at any other time also would be one gallon per minute. Thus, flow rate did not change with time. Processes that do not change with time are called "steady state."

While a process is changing with time, but is approaching a steady state, the process is called "transient." Although a process may be changing, it often does so in a smooth and continuous fashion. Consider the example of a thermometer heated to a temperature of 100°C. If temperature of liquid in the bulb of the thermometer is plotted as a function of time, a curve is obtained (Figure 1) as it cools to room temperature.

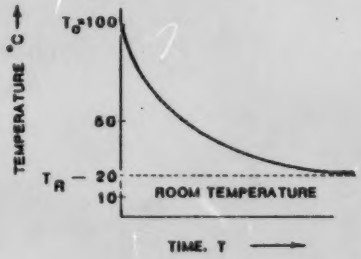



FIG. 1

When the thermometer begins to cool, the temperature difference between the liquid in the bulb and the room air is 80C°. Since heat flow rate is proportional to temperature difference, this establishes an initial heat flow rate for the system. As the thermometer cools, the temperature difference (forcelike quantity) producing the heat flow also decreases. Thus, when the thermometer bulb temperature has dropped to 60°C, the temperature difference is only 40C°; and the heat flow rate from the bulb to the air is one-half its initial value, resulting in slower cooling of the bulb. This process continues, with an ever-decreasing rate of temperature change, until the thermometer bulb reaches room temperature after some lengthy time.

The temperature of the liquid in the thermometer is given at any time t by Equation 1.

$$T = T_{amb} + [T_o - T_{amb}]\, e^{-kt} \qquad (1)$$

where

$T$ = temperature of thermometer at time t.
$T_o$ = temperature of thermometer at time t = 0.
$T_{amb}$ = ambient temperature (room temperature $T_R$ in this case).
$k$ = decay constant with dimensions of inverse time.
$e$ = exponential base = 2.71825.
$t$ = time.

When t = 0, Equation 1 reduces to —

$$T = T_R + (T_o - T_R)\, e^{-o} = T_R + (T_o - T_R)(1),$$

since $e^o$ equals unity. Finally then, we get $T = T_o$, which confirms the claim above that $T_o$ is the temperature at time t = 0. As time goes on, liquid in the thermometer approaches room temperature in a smooth and continuous fashion, as shown in Figure 1. Curves that decrease in accordance with Equation 1 are called "exponential."

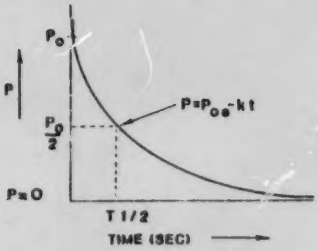



FIG. 2

The curve shown in Figure 2 indicates that some parameter P is an exponentially-decreasing function of time in accordance with the equation $P = P_o e^{-kt}$. Notice that the curve is similar to the temperature profile plotted in Figure 1.

The curve in Figure 2 begins with a peak value of $P = P_o$ at time t = 0. The curve approaches P = 0 exponentially as time increases indefinitely. Notice that P does not reach P = 0 until time $t = \infty$! But, for all practical purposes, it almost reaches P = 0 long before. Since a finite time when it reaches zero cannot be given, a convenient time interval called $T_{1/2}$, which is the time for the system to decrease from $P = P_o$ to $P = P_o/2$, is introduced. Because the initial value $P_o$ has been reduced by a factor of one-half, the time interval $T_{1/2}$ is called the "half-life" for the changing parameter P. Observe carefully the values of $P_o/2$ and corresponding point $T_{1/2}$ in Figure 2. Half-life $T_{1/2}$ is a convenient time interval — referred to as a "time constant" — for gaging rate of change in systems that follow an equation such as $P = P_o e^{-kt}$. It often is used in nuclear systems in which radioactive decay, an exponentially-decaying process following an equation like $P = P_o e^{-kt}$, is studied.

The time required for a system to change from any P to P/2 is indicated by Equation 2.

$$T_{1/2} = \frac{0.693}{k} \qquad (2)$$

where

k = constant in equation of the type $P = P_o e^{-kt}$.
$T_{1/2}$ = time required for system to change from any P to P/2.

Another commonly-used time constant is defined as the time interval during which the initial parameter $P_o$ decreases to $P_o/e$ (rather than to $P_o/2$), where e is the naperian base = 2.71828.

This time constant, denoted $T_{1/e} \equiv \tau$, is equal to $1/k$, where $k$ is the same constant defined earlier.

$$T_{1/e} \equiv \tau = \frac{1}{k} \qquad (3)$$

The value for $\tau = 1/k$ easily is obtained as follows:

$$P = P_o e^{-kt}$$

(1) Set $P = \frac{P_o}{e}$ at $t = \tau$: $\quad \frac{P_o}{e} = P_o e^{-k\tau}$

(2) So that — $\quad \frac{1}{e} = e^{-1} = e^{-k\tau}$

(3) This arrangement requires — $\quad -1 = -k\tau$ or $\tau = \frac{1}{k}$.

Since $\tau = 1/k$ is the time for $P = P_o$ to decrease to $P = P_o/e$, and $P_o/e = P_o/2.718 \simeq 0.37\ P_o$, $\tau$ is the time for $P_o$ to decrease to $0.37\ P_o$. The time constant $T_{1/2}$, discussed previously, reduces $P_o$ to $0.50\ P_o$.

Times at which the parameter $P_o$ reached $1/2\ P_o$ or $0.37\ P_o$ are shown in Figure 3. Either time constant could be used to characterize the rate at which the parameter is decreasing.

Example A illustrates use of $T_{1/2}$ and $\tau$ in a system undergoing an exponential decrease.

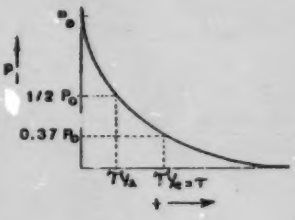



FIG. 3

**EXAMPLE A: HOT BODY COOLING TOWARD ROOM TEMPERATURE**

Given: A hot block of iron cools according to the equation $T = T_R + (T_o - T_R)\ e^{-kt}$, where $T_R = 20°C$, $T_o = 700°C$ and $k = 0.04$/min.

Find:
(a) What time constant $T_{1/2}$ equals and temperature at that time.
(b) What time constant $T_{1/e} = \tau$ equals and temperature at that time.
(c) Final temperature after a time interval equal to $5\ \tau$ (that is, $t = 5\ \tau$) has elapsed.

Solution: (a) $T_{1/2} = \frac{0.693}{k} = \frac{0.693}{0.04/\text{min}} = 17.3$ min

After 17.3 min, temperature $T = \frac{T_o}{2} = \frac{700}{2} = 350°C$

(b) $\tau = \frac{1}{k} = \frac{1}{0.04/\text{min}} = 25$ min

After 25 min, temperature $T = \frac{T_o}{e} = 257.5°C$

(c) $T = T_R + (T_o - T_R)\, e^{-kt}$

$t = 5\,\tau = 5\,(25) = 125$ min

$T = 20 + (700 - 20)\, e^{-(0.04)(125)}$

$= 20 + 680\, e^{-5}$

$= 20 + 680\,(0.0067) = 20 + 4.6$

$T = 24.6°C$

Thus, after five time constants or two hours and five minutes, temperature of the hot body has dropped from 700°C to 24.6°C. It still has ≈ 5°C to go to reach 20°C. This change will require a long time, since the decrease is now very slow. However, note that, in five time constants, temperature already has dropped 675°C of the 680°C it has to go to reach room temperature at 20°C. This is more than 99% of the total change.

In a similar manner, an exponentially-increasing function is shown in Figure 4. The changing parameter q begins at zero time t = zero and rises toward Q as time increases; again the time constant $\tau = 1/k$ is useful in describing the process. Since $q = Q\,(1 - e^{-t/\tau})$, $q = Q\,(1 - e^{-1})$ when $t = \tau$. Thus, the parameter q has reached $q = Q\,(1 - 0.37) = 0.63\,Q$, or 63% of its final value Q after one time constant.

The parameter q never reaches its maximum value Q, just as the decreasing P parameter never reaches zero. One can show that both exponentially-increasing and decreasing functions reach 99% of their final values after 5 time constants, $t = 5\,\tau$, as was shown in Example A.

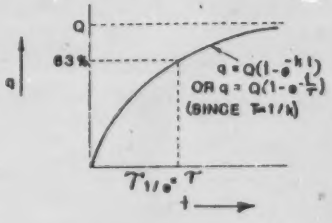



FIG. 4

## RADIOACTIVE DECAY

Radioactive decay is spontaneous disintegration or transformation of one nuclide into another. Radioactive decay results in a decrease, with time, in the number of original radioactive atoms in the sample, because those radioactive nuclides that undergo decay usually are changed into more stable types of nuclides. To determine exactly when one particular radioactive atom will decay is impossible, because the decay process is random. However, because the total number of original radioactive atoms present in any sample is extremely large, this number undergoes a predictable, continuous change. The number of atoms undergoing decay in any specified time period, and the amount of radioactivity produced, can be calculated with reasonable accuracy.

An important property of a radioactive nuclide is its half-life. Half-life is the time in which half the atoms of a particular radioactive nuclide decay to another nuclide. Each radioactive nuclide has its own characteristic half-life.

Assume that one has a radioactive source and that some means, such as a Geiger counter, of counting the number of atoms that disintegrate in a given time interval is available. If one plots the number of original atoms present at a given time versus time on a linear graph (Figure 5), the curve obtained indicates an exponential, or logarithmic, relationship. If the number present versus time is plotted on semilog paper (Figure 6), a straight line is obtained, which, again, indicates that radioactive decay is an exponential process. When there is a constant fractional decrease in the parameter during equal intervals of time, decay is exponential. Although the same number of atoms do not disintegrate during each unit of time, the same fraction of the atoms present decay during each unit of time. This behavior leads to the general exponential relationship for radioactive decay illustrated in Equation 4 and Example A.

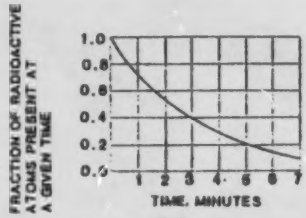



FIG. 5

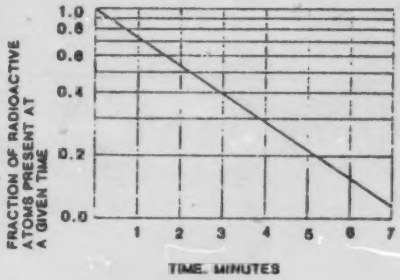



FIG. 6

$$N = N_o e^{-\lambda t} \qquad (4)$$

where

$N$ = number of undecayed atoms left at time $t$.
$N_o$ = number of atoms originally present at $t = 0$.
$\lambda$ = decay constant, $s^{-1}$, $min^{-1}$, $hour^{-1}$, etc.

Example B illustrates use of Equation 4 in a realistic radioactive-decay problem.

**EXAMPLE B: RADIOACTIVE DECAY RATE**

Given: A radioactive sample contains $10^{22}$ atoms; the decay constant is 0.1 $hr^{-1}$.

Find: (a) Number of original radioactive atoms remaining 4.0 hours from now.
(b) Time constant $\tau$.

Solution: (a) $\lambda = 0.1\ hr^{-1}$

$N_o = 10^{22}$ atoms

$t = 4$ hours

$N = N_o e^{-\lambda t}$

$= (10^{22}\ \text{atoms}) e^{-0.1\ hr^{-1} \times 4.0\ hr}$

$= 10^{22}\ \text{atoms}\ e^{-0.4}$

$= 0.67 \times 10^{22}$ atoms

$N = 6.7 \times 10^{21}$ atoms

(b) Time constant $\tau = \frac{1}{k} = \frac{1}{\lambda}$

Therefore — $\tau = \frac{1}{0.1/hr} = 10$ hours

After 50 hours (five time constants), 99% of the atoms will have decayed, and only 1% will remain.

The activity of a given sample usually is more important than the number of atoms present. Activity of a sample is defined by Equation 5 and illustrated by Example C.

$$A = \lambda N \qquad (5)$$

where

A = activity, counts per minute (c/m) or disintegrations per second (d/s).
$\lambda$ = decay constant, $s^{-1}$, $min^{-1}$, $hour^{-1}$, etc.
N = number of undecayed atoms.

**EXAMPLE C: DISINTEGRATION OF A RADIOACTIVE MATERIAL**

**Given:** At a given instant of time t, a radioactive sample contains $10^{18}$ atoms and has a decay constant of $10^{-7}$ $s^{-1}$.

**Find:** Activity in disintegrations per second at that time.

**Solution:** $N = 10^{18}$ atoms at time t

$\lambda = 10^{-7}\ s^{-1}$

$A = \lambda N$

$= 10^{-7}\ s^{-1}\ 10^{18}$ atoms

$= 10^{11}$ atoms/s

$A = 10^{11}$ disintegrations per second

Notice that activity at any time depends on number of undecayed atoms remaining at that time, and so must decrease with time.

Decay constant $\lambda$ is closely related to concept of half-life of a radioisotope. Half-life, $T_{1/2}$, is defined as time required for a given isotope to decay to one-half its initial quantity or activity. Consider an initial activity of $A_o$. At some time $t = T_{1/2}$, activity will be equal to $A_o/2$. Relationship for activity or disintegration rate A as a function of time is similar to the relation for number of atoms N as a function of time, given by $N = N_o e^{-\lambda t}$ (Equation 4). Thus, activity A has this form:

$$A = A_o e^{-\lambda t}$$

After one half-life, $t = T_{1/2}$, $A = A_o/2$ so that the following is derived:

$$\frac{1}{2}A_o = A_o e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Take the natural log of both sides:

$$\ln\left(\frac{1}{2}\right) = -\lambda T_{1/2}$$

$$\lambda T_{1/2} = 0.693$$

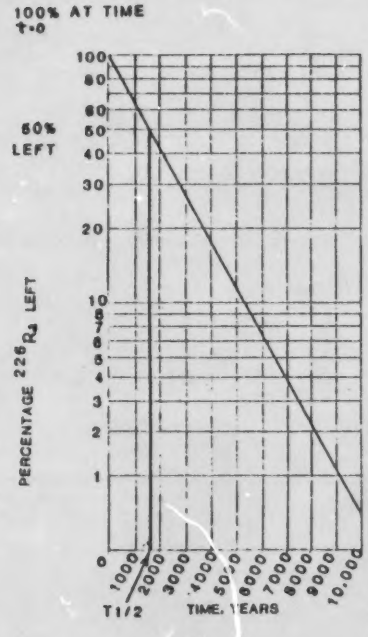



FIG. 7

Rearrange and solve for $T_{1/2}$:

$$T_{1/2} = \frac{0.693}{\lambda}$$

This relationship is similar to development for $\tau = 1/k$ (Equation 2). Measured half-lives range from about $10^{-14}$ years ($3 \times 10^{-7}$ s) for $^{212}Po$ to about $10^{7}$ years for $^{209}Bi$. A decay curve for radium-226 indicating a half-life of 1600 years is illustrated in Figure 7.

## MECHANICAL SYSTEMS

### TRANSLATIONAL

If a mass is attached to a spring and started in vertical motion (left portion of Figure 8), displacement of the mass as a function of time will result in a graph like that shown in Figure 8.

The affect of friction usually is neglected in a first discussion of this motion. Clearly, if no friction is present, vertical vibration will continue indefinitely. Actually, a weight on a spring will not oscillate indefinitely; since friction always is present, amplitude will decrease regularly, and the mass eventually will come to rest. The motion is said to be damped out by friction and is called "damped motion." How fast the weight comes to rest depends upon how much it is damped.

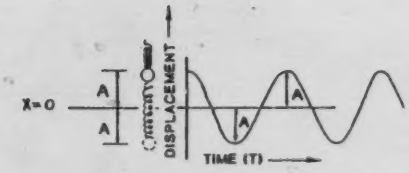



FIG. 8

If some small amount of friction, such as air drag, is present, the displacement-time graph shown in Figure 8 resembles that shown in Figure 9.

The time constant $T_{1/2}$ of this system is defined as time required for the amplitude of the oscillations to decrease from $x_o$ to $x_o/2$. The time constant $T_{1/e} = \tau$ is defined as time required for the amplitude of the oscillations to decrease to $x_o/e$ or $0.37\ x_o$. $T_{1/2}$ and $\tau$ are illustrated in Figure 9.

## ROTATIONAL

The balance wheel of a watch represents a mechanical system in which oscillation is damped due to frictional torques. The balance wheel of a watch that was originally at position $\theta = 0°$ at time $t = 0$ is rotated to $\theta = 0.1$ rad (57.3°) and released. The wheel then oscillates back and forth around the position $\theta = 0°$. If angular displacement of the wheel is plotted as a function of time, the displacement-time curve appears as shown in Figure 10. (Notice carefully that for this motion, displacement is measured in radians or degrees.) The time constant $T_{1/2}$ of this balance wheel is defined as time required for the initial angular displacement $\theta = 0.1$ rad to damp out (due to frictional torques) to 1/2 this initial value, or 0.05 rad. The time constant $\tau$ for this motion is time for the initial amplitude of 0.1 rad to damp out to a value equal to 0.37 (0.1 rad) or 0.037 rad. Both of these values are shown in Figure 10.

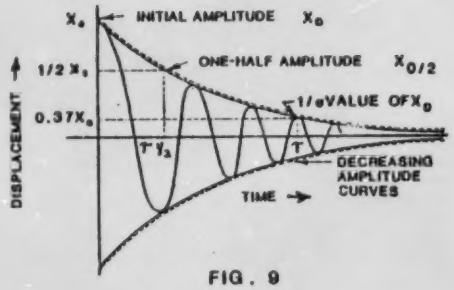



FIG. 9

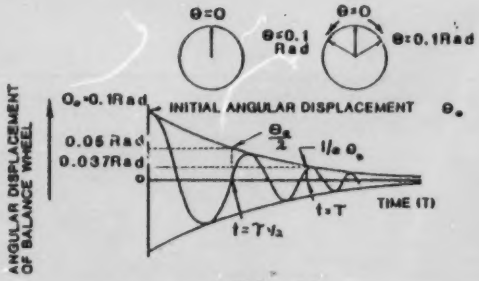



FIG. 10

## FLUIDAL SYSTEMS

Time constants are not often applied in fluidal systems, but are very useful in one specific problem. This problem concerns the time required to empty a tank because of the influence of gravity. Figure 11 illustrates such a problem. The flow rate from the tank depends upon the pressure at the bottom of the tank, which, is turn, depends upon the height of the liquid above the opening. At $t_1$ of Figure 11a, the height $h_1$ produces a certain flow rate. At $t_2$ the height has decreased to $h_2$, and the flow rate from the tank has decreased proportionally. Thus, the liquid level drops as an exponential curve, as shown in Figure 11b.

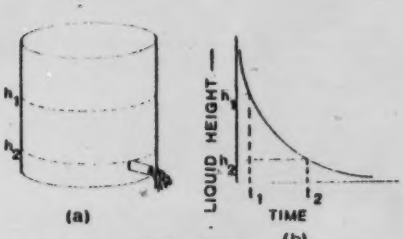



FIG. 11

## ELECTRICAL SYSTEMS

Now let us consider the process of charging and discharging a capacitor. We shall consider the two special cases illustrated in Figure 12.

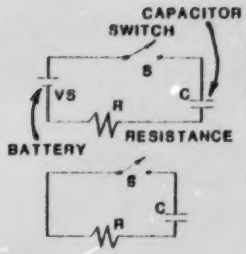



FIG. 12

In Figure 12a, the switch is ready to be closed with the capacitor completely uncharged. In Figure 12b, the capacitor already is charged to a voltage $V_s$, and the closing of the switch short circuits the capacitor; thus, 12a illustrates a charging circuit and 12b is a discharging circuit. R represents total resistance of the circuit.

When the switch is closed in Figure 12a, voltage across the capacitor as a function of time is given by Equation 6.

$$V_c = V_s\ (1 - e^{-t/RC}) \qquad (6)$$

where

$V_c$ = voltage across capacitor.
$V_s$ = voltage to which capacitor is charged.

Charging current as a function of time is given by Equation 7.

$$I = \frac{V_s}{R} e^{-t/RC} \qquad (7)$$

where

I = charging current.
R = resistance.

Charging current and voltages are shown in Figure 13. Comparing Equation 7 with the general form $P = P_o e^{-kt}$, we see that $k = 1/RC$. Then, since $\tau = 1/k$ in general, $\tau$ must equal RC for Equation 7. Thus, setting $RC = \tau$ in Equations 6 and 7 yields Equation 8.

$$V_c = V_s\ (1 - e^{-t/\tau}) \qquad (8)$$

where

$\tau = RC$ = time constant.

At a time equal to one time constant, Equation 8 becomes —

$$V_c = V_s\ (1 - e^{-1}), \text{ and } I = \frac{V_s}{R} e^{-1}$$

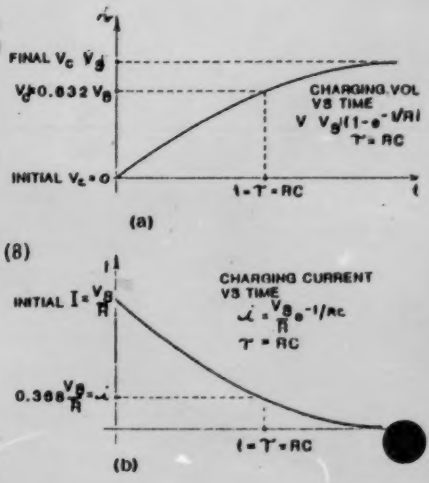



FIG. 13

$$V_c = V_s\ (1 - 0.368), \text{ and } I = \frac{V_s}{R}\ (0.368)$$

or

$$V_c = 0.632\ V_s, \text{ and } I = 0.368\ \frac{V_s}{R}$$

In other words, at the end of one time constant $\tau = RC$, voltage across the capacitor is 63% of supply voltage, and current has dropped to 36.8% of its original value.

During the discharge phase shown in Figure 12b, voltage across the capacitor and the circuit current are given by the graphs shown in Figures 14a and 14b, respectively. As before, the time constant is $\tau = RC$. As indicated in Figure 14, after a time equal to one time constant $\tau = RC$, voltage decreases from $V_s$ to 37% of $V_s$, while current decreases from a maximum magnitude of $V_s/R$ to 37% of $V_s/R$. (The negative sign indicates that discharge current flows in opposite direction to the direction previously assigned to charging current.) Example D illustrates how the time constant is used in practical problems.

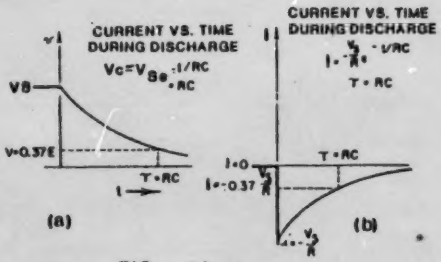
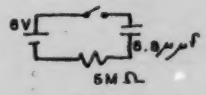



FIG. 14

**EXAMPLE D: TIME CONSTANT CALCULATION IN AN RC CIRCUIT**

**Given:** From Figure 12a, resistance $R = 5\ M\Omega$, and capacitance $C = 6.8\ \mu\mu f$, and $E = 6\ V$.




**Find:** Time constant of this circuit. What does this time constant indicate?

**Solution:** $T = RC = 5 \times 10^6 \times 6.8 \times 10^{-12} = 34.0 \times 10^{-6}$ s, or

$T = 34\ \mu s$.

This value means that voltage across the capacitor reaches 63.2% of 6 V or $v_c = 3.792$ V after 34 μs. It also means that current at the same time has a value of —

$$I = \frac{V_s}{R}\ (0.368) = \frac{6\ V}{5 \times 10^6}\ 0.368 = 0.44 \times 10^{-6} \text{ amps, or}$$

$I = 0.44$ μamps

## THERMAL SYSTEMS

Hot objects cool down to the temperature of their surroundings. If temperature difference $\Delta T$ between an object and its surroundings is not too great, the rate of cooling is approximately proportional to temperature difference between the object and its surroundings. This rate results in the relationship given by Equation 9.

$$\Delta T = \Delta T_o e^{-kt} \qquad (9)$$

where

$\Delta T$ = temperature difference between object and its surroundings at time t.
$\Delta T_o$ = temperature difference between object and its surroundings at time t = 0.
k = constant.
t = time.

(Note: At this point, this equation is identical to Equation 1.)

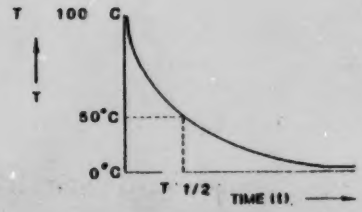



FIG. 15

A mercury-in-glass thermometer is placed in boiling water for a few minutes and then removed. Assume that temperature of the surroundings is at the freezing point, 0°C. Temperature indicated on the cooling thermometer is plotted as a function of time in Figure 15.

Since surrounding temperature is taken as 0°C, the time constant $T_{1/2}$ of this system is defined as time required for the thermometer to reach 50°C or one-half the boiling temperature of water, 100°C. This is shown in Figure 15. The student similarly should indicate the time constant $\tau$ and its meaning on Figure 15.

## OPTICAL SYSTEMS

Phosphorus used to coat a TV screen gives off light that decays at a certain rate. When the TV set is turned off, the screen continues to glow for several minutes. If intensity of light from a fluorescent or phosphorescent screen were measured and plotted as a function of time, a curve similar to that in Figure 16 would be obtained.

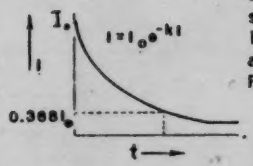



FIG. 16

Equation 10 is for light intensity.

$$I = I_o e^{-kt} \qquad (10)$$

where

$I$ = intensity at time $t$.
$I_o$ = intensity at time $t = 0$.
$k$ = decay constant.

Since this is a standard exponentially-decreasing function, the time constant in this case must be the following:

$$\tau = T_{1/e} = \frac{1}{k}$$

At this time, light intensity has dropped to 36.8% of its original brightness. After five time constants (5 $\tau$), less than 1% of the original intensity remains; therefore, the screen is almost dark. Characteristic time constants for phosphorescent decay are in the order of milliseconds to microseconds.

## SUMMARY

Exponential change of a quantity, whether increasing or decreasing, is described conveniently by use of a time constant for the process. Two such time constants are in common use, and both have been described in this module. In either case, a small time constant for the system characterizes a relatively rapid change, and a large time constant characterizes a relatively slow change.

The time constant provides another quantity that finds application in all basic physical systems. In any system, when a quantity is changing and when its rate of change is proportional to its present value, an exponential change occurs. In this module, exponential changes and corresponding time constants have been shown to apply to the following systems:

- Mechanical-translational system, in which rate of change of amplitude of linear oscillations is proportional to amplitude of any time.
- Mechanical-rotational system, in which rate of change of angular displacement of oscillations is proportional to angular displacement at any time.
- Electrical system, in which rate of change of voltage (or current) on a charged capacitor is proportional to voltage (or current) at any time.

- Thermal system, in which rate of change of temperature difference of a body with its ambient temperature is proportional to temperature difference at any time.

- Optical system, in which rate of change of light intensity is proportional to time.

- Nuclear system, in which rate of disintegrations of radioactive atoms is proportional to number of radioactive atoms present at any time.

These cases are a small sample of similar situations in physics. An understanding of exponential change in terms of a time constant helps the technician to unify all such cases by abstracting from such diverse physical processes their essential similarity in behavior.

## MECHANICAL SYSTEM - LINEAR TRANSLATIONAL

### MATERIALS

Stopwatch.
Spring with weight (spring should be quite flexible to give a large amplitude).

### PROCEDURE

1. Suspend a weight from a flexible coiled spring, as shown in Figure 17.

2. Indicate, on a meter stick, a positive 10 cm above and 10 cm below equilibrium position by symbols $+x_o$ and $-x_o$. This distance does not have to be 10 cm; but the larger the displacement value, the easier the measurements will be. Also label positions $+0.5\ x_o$ and $-0.5\ x_o$.

3. Move weight up to position $+x_o$, and release weight. Be careful not to give weight any side-to-side motion.

4. Start stopwatch at instant the weight is released, and carefully note times when oscillations reach $\pm 0.9\ x_o$, $\pm 0.8\ x_o$, $\pm 0.7\ x_o$, $\pm 0.6\ x_o$, $\pm 0.5\ x_o$, $\pm 0.4\ x_o$, $\pm 0.3\ x_o$, $\pm 0.2\ x_o$, $\pm 0.1\ x_o$, etc.

5. Plot data as shown in Figure 18.

6. On Figure 18, indicate time when value of oscillation reached 1/2 its original value and label this time $T_{1/2}$. What is experimental value of $T_{1/2}$?

7. In a time equal to $5\ T_{1/2}$, amplitude should have decreased to about $0.03\ x_o$. Calculate $5\ T_{1/2}$, and check whether or not it is greater than time required to decrease amplitude to $0.1\ x_o$. (It should be!)

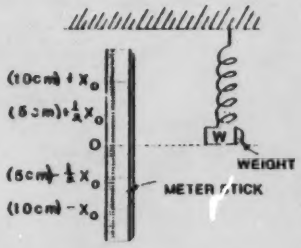



FIG. 17

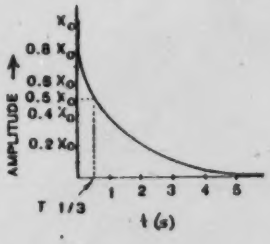



FIG. 18

## MECHANICAL SYSTEM - ROTATIONAL

### MATERIALS

Torsion pendulum flywheel on shaft.

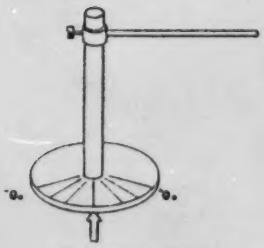



FIG. 19

### PROCEDURE

1. Suspend a circular weight on a stiff shaft, as shown in Figure 19.
2. Rotate disc to a predetermined angle $\pm\theta_0$, and carefully release disc.
3. Record the time required for amplitudes of oscillation to reach several intermediate angles before coming to rest. Record time here: _____.
4. Plot this data as in Figure 20.
5. Draw a horizontal line from 0.5 $\theta_0$ until it intersects the curve. Draw a vertical line from this point to time axis. Read time constant $T_{1/2}$ off graph.
6. When will the oscillative disc have decreased its amplitude of oscillation to a value of 3% of its original amplitude?

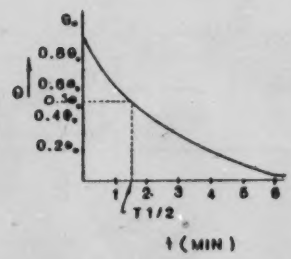



FIG. 20

# THERMAL SYSTEM

## MATERIALS

Thermometer.
Beaker.
Bunsen burner.

## PROCEDURE

1. Bring a container of water to boil.
2. Note room temperature.
3. Place a thermometer in water, and let it come to equilibrium.
4. Remove thermometer, and immediately begin reading temperature versus time at 5°C intervals.
5. In Data Table, record temperature versus time.
6. Plot data obtained as in Figure 21.
7. Determine time constant $T_{1/2}$ and $\tau = T_{1/e}$ for this thermometer.

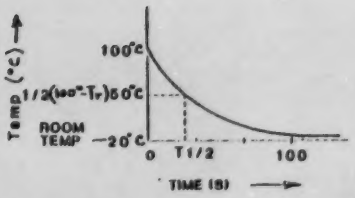



FIG. 21

| DATA TABLE | | | |
|---|---|---|---|
| Temperature °C | Time (s) | Temperature °C | Time (s) |
| 100 | 0 | 60 | |
| 95 | | 55 | |
| 90 | | 50 | |
| 85 | | 45 | |
| 80 | | 40 | |
| 75 | | 35 | |
| 70 | | 30 | |
| 65 | | 25 | |
| | | $T_R$ = 20 | |

## ELECTRICAL SYSTEM

### MATERIALS

6-volt battery.
Stopwatch.
Resistors (15 kΩ and 25 kΩ).
Capacitor (2000 μf - electrolytic).
Switch (DPDT).
Microammeter (full-scale; at least 400 microamps).

### PROCEDURE

1. Connect circuit elements as shown in Figure 22 to terminals of a double-pole, double-throw (DPDT) switch. CAUTION: Since capacitor is electrolytic, polarity must be observed. Have instructor check wiring.

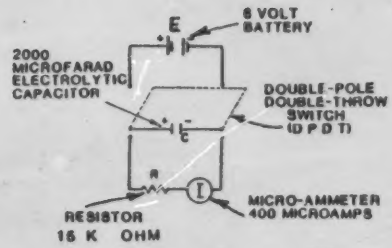



FIG. 22

2. Use 15-kΩ resistor provided. Charge capacitor for a few seconds by throwing switch toward battery-connected terminals. Discharge capacitor by reversing throw of switch, at same time starting time measurement with stopwatch. Take data on current read by meter versus time elapsed. If current decreases too rapidly to make all measurements during one discharge, process can be repeated, each time measuring one or more current-versus-time data points.

3. Repeat for a 25-kΩ resistor.

4. Plot current versus time and, from graph for each resistor, determine time constant $\tau$. How does experimental value compare with theoretical?

## OPTICAL SYSTEM

### MATERIALS

Hook-up wire.
TV or oscilloscope.
Optical power meter.
Strip chart recorder.

**PROCEDURE**

1. Turn on TV set or oscilloscope, and allow sufficient time for warm-up (showing a bright picture tube).

2. Place optical power meter in front of screen, as shown in Figure 23.

3. Record output of optical power meter as a function of time on a chart recorder (Figure 24). Be sure to leave power meter at same position near TV set while taking readings. Do not change ambient lighting conditions while readings are being taken.

4. Determine time required for intensity of fluorescence to reach 50% of its peak value after turning it off.

5. Determine time constant $T_{1/2}$.

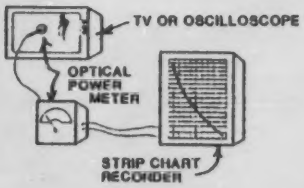



FIG. 23

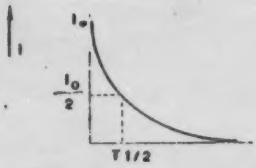



FIG. 24

## SELF CHECK

1. Define the following terms:

   Steady state.
   Transient.
   Damping.
   Time constant.
   Half-life.
   Decay constant.

2. Draw a graph for an exponentially decaying quantity P. Show on the graph the time constants $T_{1/2}$ and $\tau$ and the values of P at these times. Write an equation for the graph.

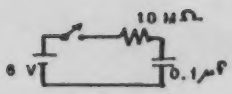
3. A hot body at 500°C is placed in a cooler at 0°C. After five minutes, the temperature is 400°C. Determine the following:

   a. Decay constant k.
   b. Time constant $\tau$.
   c. Temperature after two time constants ($t = 2\tau$).
   d. Temperature after five time constants ($t = 5\tau$).

4. The common isotope of uranium $^{238}U$ has a half-life of $4.50 \times 10^9$ years. Determine the following:

   a. Decay constant $\lambda$.
   b. The fraction of original atoms remaining after $9.0 \times 10^9$ years.
   c. The number atoms of $^{238}U$ needed for an activity of 1 curie. (A curie is defined as an activity of $3.70 \times 10^{10}$ d/s.)

5. Given the circuit shown in the adjoining sketch, determine the following:




   a. Time constant.
   b. The current through the resistor when the switch is closed.
   c. The current through the resistor after two time constants.
   d. The voltage on the capacitor after three time constants.
   e. The voltage on the capacitor after five time constants.

6. Explain the use of time constants in each of the following energy systems, and give an example in each system:

   Mechanical translational.
   Mechanical rotational.
   Fluidal.
   Electrical.
   Thermal.
   Optical.
   Nuclear.

## REFERENCES


Aldridge, Bill B. Quantitative Aspect of Science and Technology: An Introduction. Columbus, OH: Charles E. Merrill Publishing Co., 1967.

Benedict, Robert P. Fundamentals of Temperature, Pressure, and Flow Measurements. 2nd ed. New York: John Wiley & Sons, 1977.

Bursh, Vannemar; Handley, George B.; and Timble, William H. Principles of Electrical Engineering. New York: McGraw-Hill, 1956.

Halliday, David and Resnick, Robert. Fundamentals of Physics. Revised Printing. New York: John Wiley & Sons, Inc., 1974.

Manning, Kenneth V.; Weber, Robert L.; and White, Marsh W. Physics for Science and Engineering. New York: McGraw-Hill, 1959.

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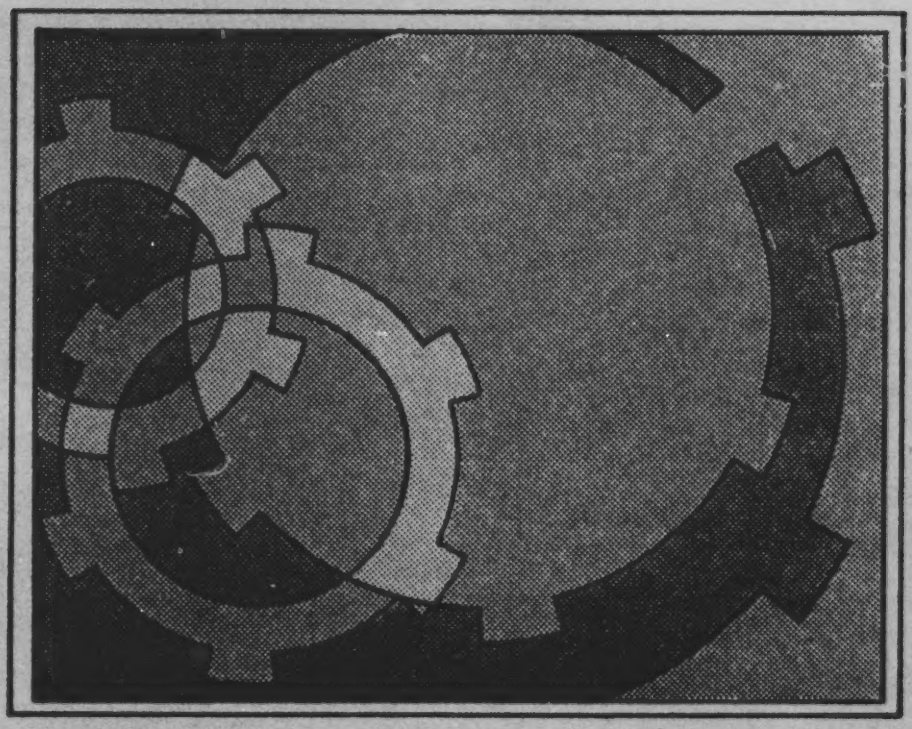
UNIFIED TECHNICAL CONCEPTS

MODULE 12 M 1

SHOCK
ISOLATION SYSTEMS



# MECHANICAL APPLICATION OF TIME CONSTANTS

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## INTRODUCTION

It is often necessary to isolate some section of a mechanical system from vibrations produced in other portions of the system. To accomplish this, some method of damping mechanical oscillations must be employed.

In this module, we will examine the principles and some of the applications of vibration isolation systems. In the laboratory, the student will measure the effectiveness of a pneumatic vibration isolation system commonly employed in scientific laboratories.

## PREREQUISITES

Student should have completed Concept Module 12-0, "Time Constants," and Application Module 10M2, "Accelerometer."

## OBJECTIVES

To complete this module, the student should be able to:

- Describe in one or two sentences two specific problems that must be solved for effective vibration isolation.
- Discuss in two to four sentences a common method of solving each of these problems.
- Measure the transmission of mechanical vibrations isolation system, given the necessary equipment.
- Measure the time constants of two pneumatically isolated systems, given the necessary equipment.

## VIBRATION ISOLATION SYSTEMS

Figure 1 shows a vibration isolation system. The purpose of such a system is to prevent mechanical vibrations present in one portion of the system from appearing in another portion. Some vibration isolation systems such as motor mounts in automobiles and shock mounts for machinery are used to confine vibrations to the isolated structure while reducing those of the support structure. Other systems such as automobile suspensions are used to prevent mechanical vibrations in the isolated structure. In either case, there are two problems that must be solved for the proper functioning of a vibration isolation system.

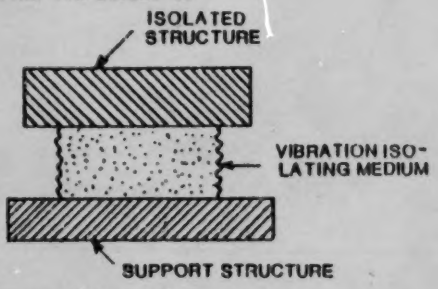



Fig. 1

1. Vibration originating in one structure must be prevented from reaching the other.
2. Any vibrations appearing in the structure that is to be vibration free, whether originating there or transmitted from the vibrating structure, must be damped as rapidly as possible.

The first problem is solved by placing a vibration isolating medium between the isolated structure and the support structure. This medium supports the mass of the isolated structure while transmitting little mechanical vibration to or from it. It is important to note that perfect isolation is not possible. Some transmitted vibration is always present.

The second problem is solved by providing damping of vibrations in the structure that is to be vibration free. This is done by making that structure as rigid as possible to reduce high frequency vibrations and by providing additional damping in the form of shock absorbers to reduce lower frequency vibrations.

In the remainder of this module, we will consider two vibration isolation systems in which the support structure may vibrate, but the isolated structure is to be vibration-free.

In an automobile suspension system, isolation is accomplished by supporting the mass of the automobile on suspension and leaf springs (Figure 2). Any mass-spring system has a preferred frequency at which it will oscillate. This preferred frequency is defined as the natural or resonant frequency. Auto-

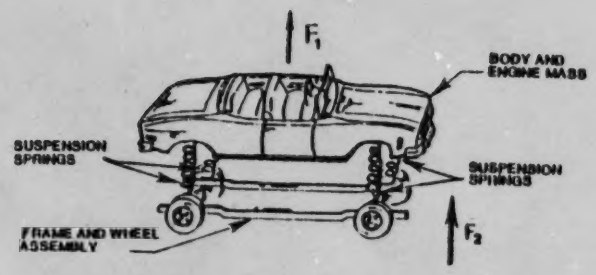



Fig. 2

mobile suspensions are designed to have low resonance frequencies. This means that forces applied to the wheels are transmitted relatively slowly through the springs to the car body. When the car hits a bump, the large upward force $F_1$ applied to the wheels has a duration time much less than the period corresponding to the resonant frequency of the suspension system. Thus, the mass of the automobile cannot react sufficiently fast and as a result, only a small fraction ($F_2$) of the original force is transmitted to the body of the car.

In the automobile, the second problem appears as vibrations at the resonance frequency that are transmitted to the mass of the car. Unless some method is employed to damp this vibration, the car will oscillate vertically for several seconds. This problem is solved with hydraulic shock absorbers designed to damp oscillations at or near the resonance frequency of the spring-mass system (Figure 3). It consists of a piston with small holes that move inside a cylinder filled with a liquid. Whenever a force is applied to the piston, some of the liquid must flow through the holes from one side of the piston to the other. The pressure exerted on the piston by the liquid damps out the vibrations of the spring-mass suspension system.

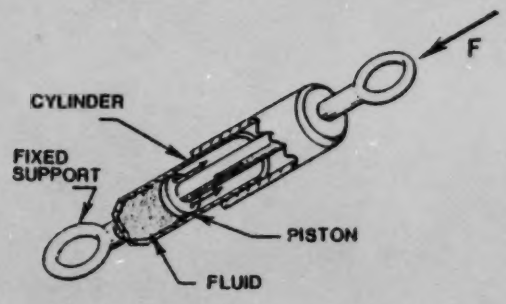



Fig. 3

Figure 4 shows the components of a pneumatic vibration isolation system commonly employed in laboratories to provide a vibration-free work surface. In this system a piston is sealed into a cylinder by a flexible diaphragm. Air pressure in the cylinder supports the piston. Damping at low frequency is provided by pressure changes as air passes through the small hole in the upper section of the cylinder. Thus, the air piston provides both isolation from floor vibrations and damping for low frequency oscillations of the table top. However,

such a system will not damp high frequency oscillations in the table top.

Figure 5 shows the results of high frequency vibration that is most troublesome in vibration isolated table tops. Although the amplitude of such vibrations is usually small, they must be eliminated for such applications as optical measuring tables, laser devices and metrology granite tables. This may be done by making the table top as heavy and as rigid as possible. For this reason, quality vibration isolation tables are of solid granite or a metal honeycomb. Another (less effective) method is to construct the table top with a layer of material that provides damping for high frequency oscillations.

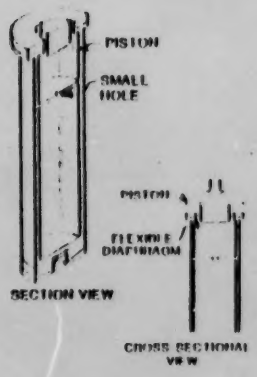



Fig. 4

## MEASUREMENT OF VIBRATION ISOLATION

The quality of vibration isolation provided by a system may be evaluated by measuring the amount of vibration transmitted through the isolating medium and the time constant for resonance frequency of the isolated mass. Both of these should be as small as possible.

The transmission of a vibration isolation system may be measured by subjecting support structure to a shock while monitoring accelerations on both support structure and isolated mass. The transmission of the system is given by Equation 1.

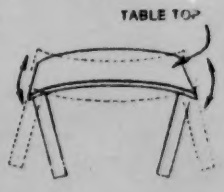



Fig. 5

$$\text{Transmission} = \frac{\text{acceleration of isolated mass}}{\text{acceleration of support structure}}. \qquad (1)$$

In symbols: $T = \frac{a_i}{a_s}$.

Time constant of the isolated mass may be measured by subjecting the mass itself to a mechanical shock while monitoring acceleration of the mass. Time constant is time required for the oscillation to drop to 1/e (37%) of its maximum value (Figure 6).

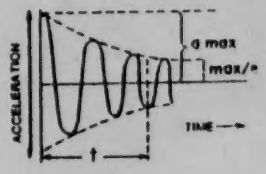



Fig. 6

## MATERIALS

Aluminum plate 1/2" x 18" x 18".
18" square of 3/8" thick plywood.
Minibike inner tube.
Golf ball.
Piezoelectric accelerometer with cable. (See Application Module 10M6, "Accelerometer.")
Aluminum block 1" x 2" x 2" drilled and tapped to accept piezoelectric accelerometer.
Oscilloscope.

## PROCEDURE

1. Attach piezoelectric accelerometer to mounting block; connect accelerometer to oscilloscope. Place aluminum plate on a table to be measured. Mark positions of points A, B, C and D of Figure 7.
2. Place accelerometer at point A with its axis vertical. Turn on oscilloscope; set it for 50 ms/cm and 0.005 volt/cm. Adjust triggering of oscilloscope so that a light touch of plate will trigger it. Note: Slight adjustments of triggering level may be necessary during this experiment. Both time scale and voltage sensitivity must be changed throughout experiment to make best observation of vibrations present.
3. Place accelerometer on table at point C. Drop golf ball from a height of 12" onto point B and observe acceleration. (Be sure to catch ball after first bounce.) Sketch vibration shown on oscilloscope in section A of data table and calculate maximum acceleration present. (See Application Module 10M6, "Accelerometer.")
4. Move accelerometer to point A and drop ball on point B. Sketch oscillations in section B and record maximum acceleration.
5. Calculate transmission of vibration from table to plate using Equation 1.
6. Fill minibike inner tube about 3/4 full of air, seal it, and place it under aluminum plate.
7. Repeat steps 3 through 5 for this isolation system. Sketch oscillations in parts C and D of data table.

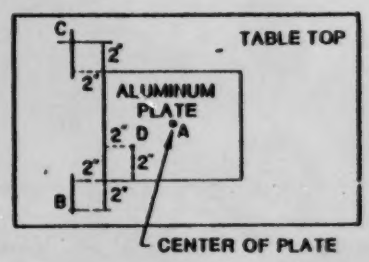


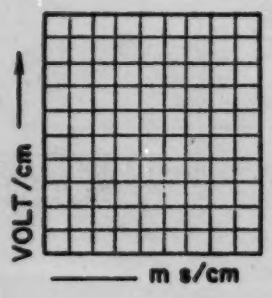

Fig. 7

8. Place accelerometer at point A and drop ball on plate at point B. Observe and sketch oscillations in part E of data table and determine time constant of the plate.
9. Place plywood square between aluminum plate and the inner tube and repeat step 8. Sketch oscillations in part F of the data table. (The wood provides high frequency damping.)

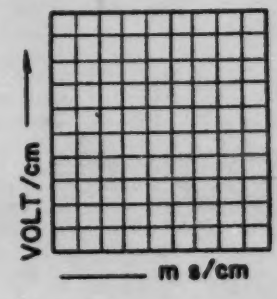
**DATA TABLE**

**NO VIBRATION ISOLATION**

Part A




Part B




Acceleration on Table.

$a_s$ = _____ g.

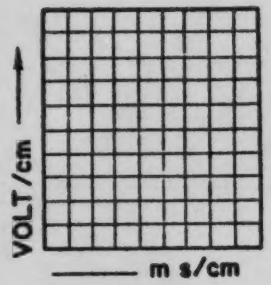
Acceleration on Plate

$a_l$ = _____ g.

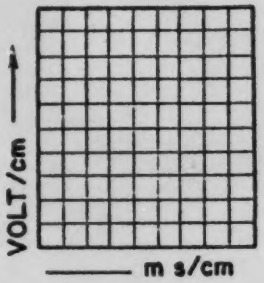
$T = \frac{a_l}{a_s}$ = _____ g ÷ _____ g = _____.

## VIBRATION ISOLATION WITH INNER TUBE

Part C




Part D




Acceleration on Table

$a_s$ = _____ g.

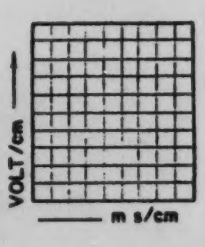
T = _____.

Acceleration on Plate

$a_l$ = _____ g.

## TIME CONSTANT WITH NO HIGH FREQUENCY DAMPING

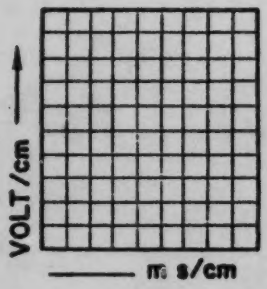
Part E




t = _____ ms.

TIME CONSTANT WITH HIGH FREQUENCY DAMPING (PLYWOOD)

Part F




t = _____ms.

## SELF CHECK

1. Describe two problems that must be solved for effective vibration isolation.
2. Describe a common method for solving each of these problems.

## REFERENCES

Reid, W.J. III. "Vibration Isolation: A Practical Understanding." From The Ealing Review (September/October, 1977).

Technical Education Research Center - SW, ed. "Optical Tables and Benches." Module 6-1, Laser and Electro-Optic Components Series developed pursuant to USOE Grant #OEG-0-8-080491-4701 (085).

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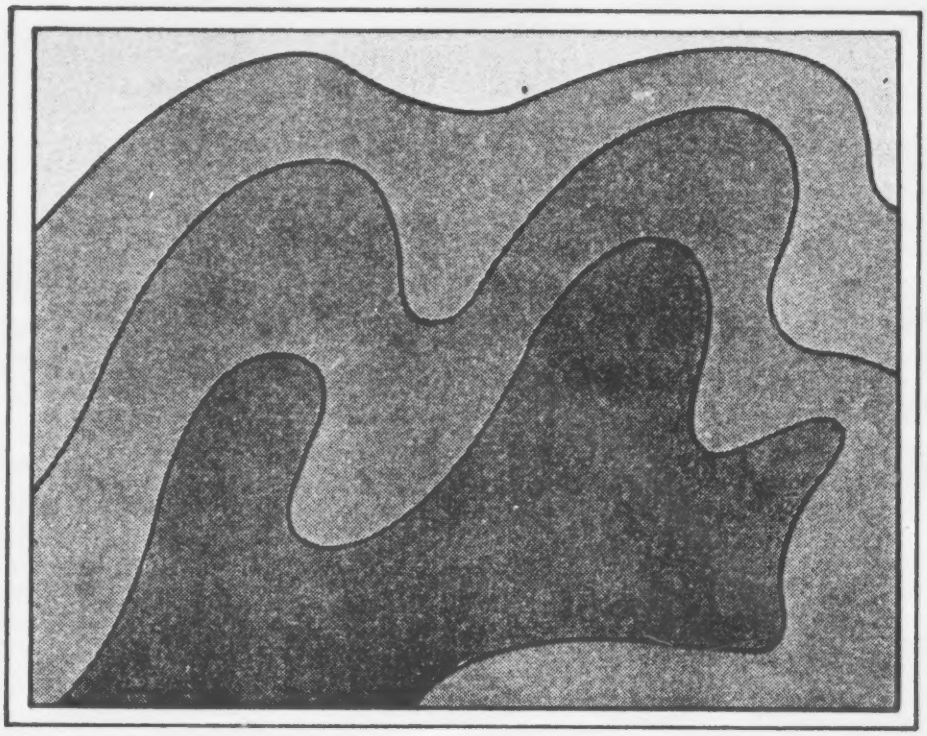
UNIFIED TECHNICAL CONCEPTS

MODULE 12 F 1

RATE OF EMPTYING TANKS



FLUID APPLICATION OF TIME CONSTANTS

Technical Education Research Center - Southwest

TERC - SW

## INTRODUCTION

Large cylindrical tanks are used widely for storage of water and other fluids in industries that handle water, waste water, chemical processing, and fire control. When these large tanks are drained by gravity, the emptying process resembles the discharge of an electrical series resistance-capacitance combination and, in a similar manner, can be described by a time constant. This module describes the process of emptying a cylindrical tank and discusses the various parameters associated with the fluid flow from the tank. In the laboratory, the student will empty a large cylindrical container to demonstrate the fluid-system behavior described in the module.

## PREREQUISITES

Student should have completed Concept Module 12-0, "Time Constants."

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Define "fluid capacitance."
- Given a fluid capacitance of a cylindrical tank and a fluid resistance of an outlet, calculate the time constant of the fluid system.
- Given the height of an initial fluid level above an outlet of a cylindrical tank and the time constant of the system, calculate the fluid level at some given time after the tank begins to empty.
- Given the time constant of an emptying cylindrical tank, calculate the approximate time required to empty the tank.
- Given the height of an initial fluid level above an outlet of a cylindrical tank, the elapsed time since the beginning of the process, and the fluid level height after the elapsed time, calculate the time constant of the tank.

- Given the appropriate equipment, determine the rate of emptying tanks.

## DISCUSSION

City water supplies usually employ large storage tanks in high places. Water is pumped into the tanks and later flows to consumers in the area. Tanks store the potential energy of the water and are, in a sense, hydraulic capacitors. (Recall: capacitors store electrical potential energy.) Tanks in need of repair must be drained before work crews go inside. Since the mass of the fluid in the tank cannot be emptied instantaneously, but requires some period of time, the necessity often arises to know the time for a transfer of fluid to take place and the manner in which the volume of fluid changes with time as the tank empties.

Figure 1 illustrates a large tank, vented to the atmosphere at the top and provided with an outlet near the bottom. The release valve behaves as a variable hydraulic resistor, since it will control the rate at which the tank will empty. (Recall: electrical resistors control the rate of charge flow.) When the valve is fully open, the tank will empty at its fastest rate. When the release valve is closed partially, the result is a larger resistance to the flow, which slows down the rate at which the tank empties. The tank itself is a fluid capacitor with stored-up (potential) fluid energy, directly proportional to the volume of fluid above the outlet.

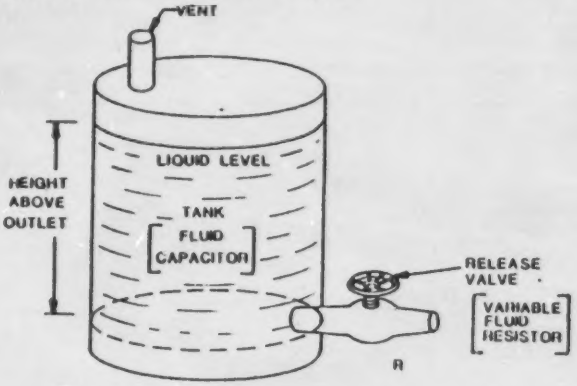



FIG. 1

The fluid capacitance can be defined by Equation 1.

$$C = \frac{V}{\Delta p} \qquad (1)$$

where

C = fluid capacitance.
V = volume of fluid stored above the outlet.
$\Delta p$ = pressure difference across the volume of fluid.

Equation 1 is analogous to the electrical definition of capacitance, which is the electrical charge stored in the capacitor divided by the electrical potential difference across the capacitor. For cylindrical tanks, the volume is the product of the height of the fluid level above the outlet and the cross-sectional area of the tank, as shown in Figure 2 and Equation 2.

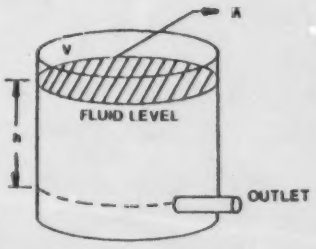



FIG. 2

$$V = Ah \qquad (2)$$

where

A = cross-sectional area of tank.
h = height of fluid level above outlet.

Pressure difference across the volume is given by Equation 3.

$$\Delta p = \rho_w h \qquad (3)$$

where

$\rho_w$ = weight density of the fluid = $\rho g$.

The capacitance of a cylindrical tank then is given by Equation 4, derived from Equations 1, 2, and 3.

$$C = \frac{V}{\Delta p} = \frac{(Ah)}{(\rho_w h)}$$

$$C = \frac{A}{\rho_w} \qquad (4)$$

**EXAMPLE A: FLUID CAPACITANCE OF A CYLINDRICAL STORAGE TANK**

Given: In a chemical plant, there is a cylindrical storage tank having a cross-sectional area of 100 $m^2$ and containing alcohol, which has a mass density of 785 $Kgm/m^3$.

Find: The capacitance of the storage tank in volume per unit pressure units.

Solution:

$$C = \frac{A}{\rho_w}$$

where

$\rho$ = mass density
g = acceleration of gravity

$$= \frac{A}{\rho g}$$

$$= \frac{100\ m^2}{(785\ Kgm/m^3)(9.8\ m/s^2)}$$

$$C = 0.013\ \frac{m^3}{\frac{Kgm}{m^3} \cdot \frac{m}{s^2}}$$

Dimensionally,

$$\frac{m^2}{\frac{Kgm}{m^3}\cdot\frac{m}{s^2}} = \frac{m^2}{\frac{\left[Kgm\cdot\frac{m}{s^2}\cdot m\right]}{m^3}} = \frac{m^2}{\left[\frac{Newton}{m^2}\right]} = \frac{m^3}{pa}$$

Remember: 1 Pascal (pa) = $\frac{Newton}{m^2}$

Therefore,

$C = 0.013\ \frac{m^3}{pa}$ ⌒ volume units
⌣ pressure units

Equation 4 shows that the capacitance of a cylindrical tank is proportional to the cross-sectional area (surface area of the liquid). Thus, a cylindrical tank with large surface area has greater fluid capacitance than a small-surface-area cylindrical tank containing fluid of the same weight density. Since the cross-sectional area of a tank (surface area of fluid) remains constant as liquid level drops in a cylindrical tank being emptied, such a tank represents a hydraulic capacitance of fixed value. Just as an electrical capacitance does not depend upon the quantity of charge stored at any time, the hydraulic capacitance of a cylindrical tank does not depend upon the quantity of fluid stored at any time.

Tanks whose cross-sectional area varies with the height of fluid level above the outlet, however, have a fluid capacitance that varies as the tank is emptied. Figure 3 illustrates this difference in different types of tanks. This module is concerned only with fixed fluid capacitors; analysis of variable fluid capacitors is beyond the scope of this module.

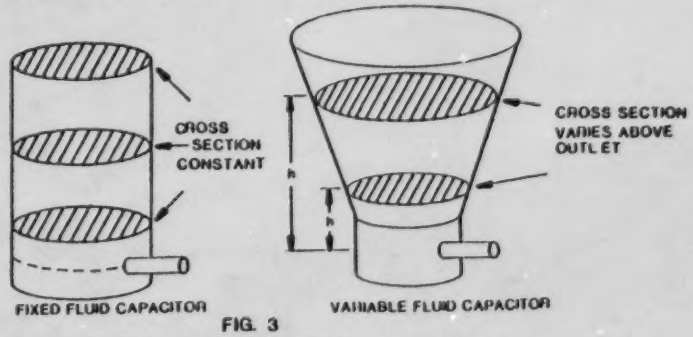



FIG. 3

The process of emptying a cylindrical tank, as illustrated in Figure 4, is analogous to the discharge of an electrical capacitor and resistor in series. Concept Module 12-0 presents the equations of discharge for such a resistor-capacitor combination. Similar equations govern the manner in which the volume flow rate of fluid varies with time as a cylindrical tank is emptied. Here the displacementlike quantity (charge) in the electrical system is

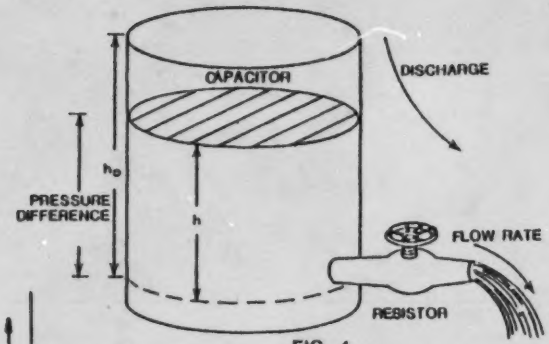



FIG. 4

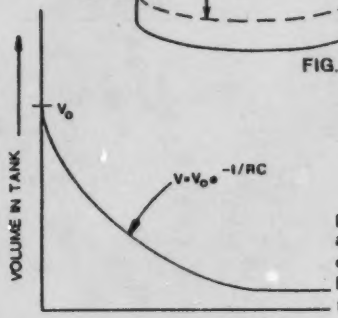



FIG. 5

replaced by the displacementlike quantity (volume) in the fluidal system. The volume remaining in the tank at any time is given by Equation 5 and plotted in Figure 5.

$$V = V_o e^{-t/RC} \qquad (5)$$

where

V = volume in tank at any time.
$V_o$ = maximum volume at time = 0.
t = elapsed time since tank began to empty.
R = fluid resistance of the outlet.
C = fluid capacitance of the tank.

Since the volume of a cylindrical tank is directly proportional to the height of the fluid above the outlet, a similar equation exists for the height of the fluid column. Equation 6 gives the height of the liquid at any known time, if the initial height, the resistance, and the capacitance of the tank are known.

$$h = h_o e^{-t/RC} \qquad (6)$$

where

h = height of liquid at any known time.
$h_o$ = initial height of liquid.
t = any known time.

The product RC is known as the "time constant." The time constant $\tau_c$ is the time required for the liquid level to drop to 0.368 $h_o$. Practically speaking, a tank would be emptied after 5 time constants of elapsed time. Equation 6 is used to solve a problem in Example B.

**EXAMPLE 8: FLUID LEVEL IN AN EMPTYING TANK**

**Given:** A water-storage tank has a fluid level 5.00 m above its outlet and a time constant of 2 hours.

**Find:**
(a) Height of the liquid level after 20 minutes.
(b) Height of the liquid level after 2 hours (one time constant).
(c) Height of the liquid level after 10 hours (five time constants).

**Solution:** The time constant $\tau_c = RC = 2$ hours (120 min).

(a) $h = h_o e^{-t/RC}$

$= (5.00\text{m}) e^{-\left(\frac{20\ \text{min}}{120\ \text{min}}\right)}$

$= (5.00\text{m}) e^{-1/6}$

$= (5.00\text{m})(0.846)$

$h = 4.23\text{m}$

(b) $h = h_o e^{-t/RC}$

$= (5.00\text{m}) e^{-\left(\frac{2\ \text{hr}}{2\ \text{hr}}\right)}$

$= (5.00\text{m}) e^{-1}$

$= (5.00\text{m})(0.368)$

$h = 1.84\text{m}$

(c) $h = h_o e^{-t/RC}$

$= (5.00\text{m}) e^{-\left(\frac{10\ \text{hr}}{2\ \text{hr}}\right)}$

$= (5.00\text{m}) e^{-5}$

$= (5.00\text{m})(0.0067)$

$h = 0.0335\text{m}$

($\sim$3.3cm — about empty)

The time constant also is the ratio of the height in the tank at any time to the height change rate, as given by Equation 7.

$$\tau_c = \frac{\text{h at time t}}{\left(\frac{\Delta h}{\Delta t}\right) \text{at time t}} \qquad (7)$$

where

$\frac{\Delta h}{\Delta t}$ = height change rate.

On the graph of Equation 6, shown in Figure 6, the time constant, therefore, is the ratio of the h at any time to the slope of the h-t curve at that time. These quantities are illustrated at two different points on the curve. This equation is useful for determining time constants graphically from an h-t plot, constructed from actual measurements during the emptying of a cylindrical tank.

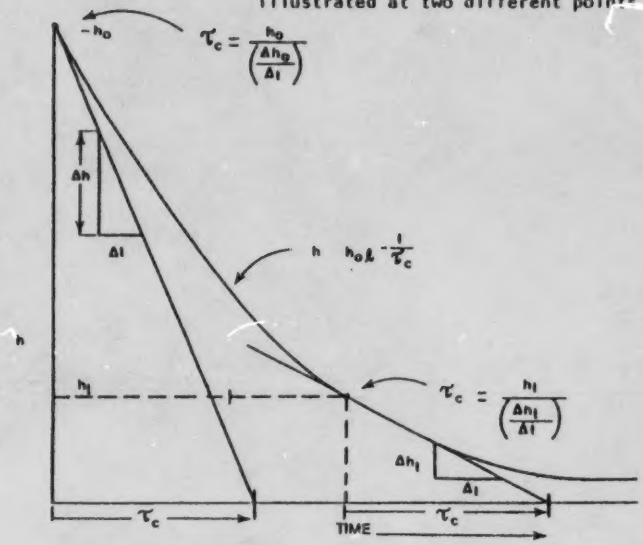



FIG. 6

In order to determine analytically the time constant of a hydraulic, capacitor-resistance, cylindrical-tank system, Equation 6 is solved for $\tau_c (=RC)$; and the result is given in Equation 8.

$$\tau_c = \frac{t}{\ln\left(\frac{h_o}{h}\right)} \qquad (8)$$

where

$h_o$ = maximum fluid height at t = 0.

h = fluid level at time = t.

t = elapsed time since emptying process began.

## EXAMPLE C: FINDING TANK PARAMETERS

**Given:** A storage tank with water of 10 m initial depth and 50 $m^2$ cross-sectional area is drained through an outlet at the bottom. After 5 minutes the fluid level has dropped to 9.5 m.

**Find:**
(a) The time constant of the tank.
(b) The capacitance of the tank.
(c) The outlet resistance.
(d) The time to drain the tank.
(e) The volume flow rate at a fluid level of 5 m above the outlet.

**Solution:** (a) Use Equation 8:

$$\tau_c = \frac{t}{\ln\left(\frac{h_o}{h}\right)}$$

$$= \frac{(5 \text{ min})}{\ln\left(\frac{10 \text{ m}}{9.5 \text{ m}}\right)}$$

$$\tau_c = 97.5 \text{ min}$$

(b) The capacitance of the system is given by Equation 4:

$$C = \frac{A}{\rho_w} = \frac{A}{\rho g} \qquad \left[\rho \text{ water} = 1000 \text{ Kgm/m}^3\right]$$

$$= \frac{(50 \text{ m}^2)}{(1000 \text{ Kg/m}^3)(9.8 \text{ m/s}^2)} = 0.0051 \left[\frac{\text{m}^3}{\frac{(\text{Kg m/s}^2)}{\text{m}^2}}\right]$$

$$C = 0.0051 \frac{\text{m}^3}{\text{pa}} \qquad \left(= \frac{\text{N}}{\text{m}^2} = 1 \text{ pa}\right)$$

(c) Since $\tau_c = RC$ —

$$R = \frac{\tau_c}{C}$$

$$= \frac{\left(97.5 \text{ m}/\text{n} \times \frac{60 \text{ s}}{\text{m}/\text{n}}\right)}{\left(0.0051 \frac{\text{m}^3}{\text{pa}}\right)}$$

$$R = 1.15 \times 10^6 \frac{\text{pa}}{\left(\frac{\text{m}^3}{\text{s}}\right)}$$

(d) Practically, $\tau_{empty} = 5\ \tau_c$:

$\tau_{empty} = 5\ (97.5\ min)$

$\tau_{empty} = 487.5\ min\ (8\ hours\ 7.5\ min)$

(e) From Equation 7 –

$$\left(\frac{\Delta h}{\Delta t}\right) = \frac{1}{\tau_c} h$$

Multiplication by the cross-sectional area (A) –

$$A\left(\frac{\Delta h}{\Delta t}\right) = \frac{1}{\tau_c} Ah$$

$$\frac{\Delta v}{\Delta t} = \frac{1}{\tau_c} Ah$$

$$= \frac{1}{(97.5\ min)}\ (50\ m^2)(5\ m)$$

$$\frac{\Delta v}{\Delta t} = 2.56\ \frac{m^3}{min} \quad \text{(volume flow rate at 5 m)}$$

Notice that since the capacitance of the cylindrical tank is a fixed value, the shape and curvature of the exponential curve of Figure 6 is determined by the value of the resistance to flow. If the resistance to flow of the outlet is small, the exponential graphs will exhibit more curvature, however, if the resistance to flow at the outlet is large, the exponential graph has such a large time constant that the exponential curve has little curvature and appears to be almost linear, as shown in Figure 7.

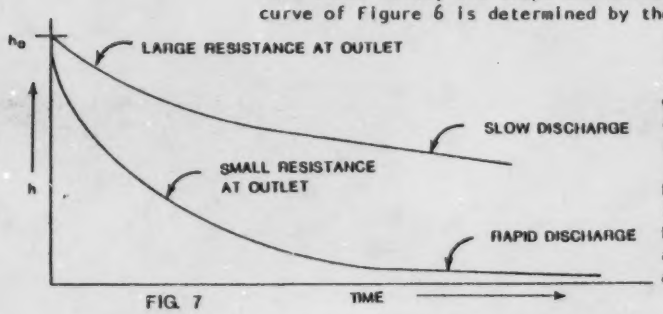



FIG. 7

## SUMMARY

The process of emptying cylindrical tanks can be thought of as the discharge of a volume of fluid through a fluid capacitor (the tank) and a resistor (the outlet) in series combination. The product of the fluid resistance and fluid capacitance of the tank is the "time constant" of the hydraulic system. In one time constant, a system discharges to approximately 37 percent of its original volume. After five time constants, the cylindrical storage tanks are practically empty.

# RATE OF EMPTYING TANKS

## MATERIALS

A large container (preferably transparent).
[Suggestion: Automatic Pipet Rinser found in Chemistry Laboratories or 1 liter Graduated cylinder with hole drilled about 2 cm above the base. Diameter of hole may range from 1/8 to 1/4 inch.]
Stop watch.
Meter stick.
A laboratory sink and water supply.
Rubber stopper to plug orifice.
Felt-tip pen or marking pencil.
Graph paper, 10 squares per inch.

## PROCEDURE

1. Arrange equipment as shown in Figure 8.

2. With a felt-tip pen, mark reference level "-0-" at level of center of orifice on side of container (tank).

3. Measure vertically with meter stick from reference "0" level. Mark 5 cm steps vertically as far as possible, as shown in Figure 8.

4. Mark vertical distance of $h_{max}$ (the maximum height available with tank full); record in Data Table 1.

5. Measure inside diameter of tank and diameter of orifice, and record these values in Data Table 1.

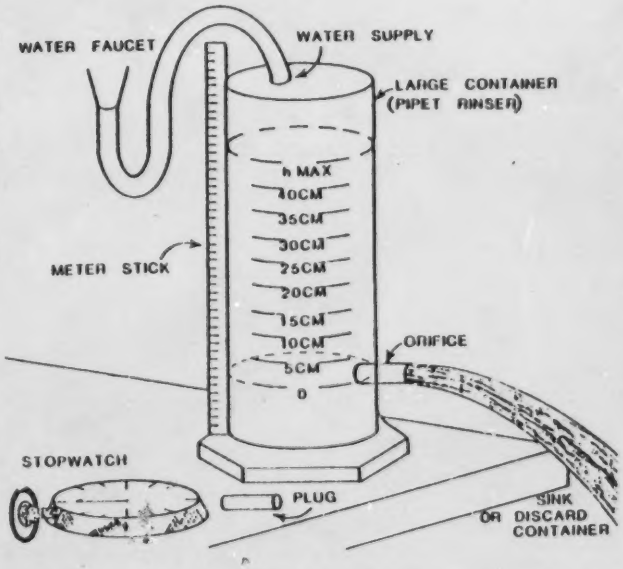



FIG. 8

6. Plug orifice tightly, and fill tank until water level is at maximum height. Shut off water supply.

7. Start stop watch when the orifice plug is removed, and measure time required to empty tank. Stop the stop watch the moment orifice ceases to discharge in a continuous flow. Record in Data Table I the time required to empty.

8. Repeat procedure of steps 6 and 7, setting liquid level at each successive 5-cm mark marked on side of container and recording in Data Table I the corresponding times to empty.

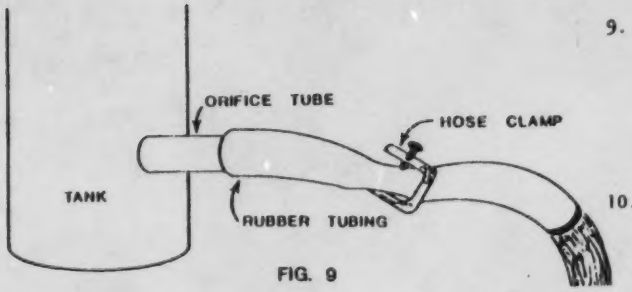



FIG. 9

9. Repeat experiment steps 6, 7, and 8 after increasing resistance to flow by attaching a short piece of rubber tubing with a hose clamp, as shown in Figure 9. Record results in Data Table 2.

10. Calculate time of fall for each level of both trials by subtracting each successive time to empty. Record in the respective columns of Data Tables I and 2.

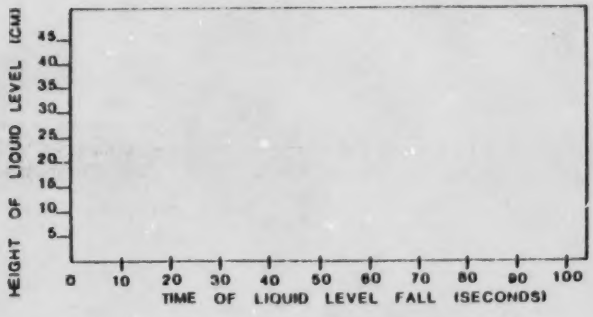



FIG. 10

11. On graph paper, plot height-of-liquid-level versus time-of-liquid-level-fall. (Lay out graph as illustrated in Figure 10. Use 10 graph-paper squares per inch.)

12. From graph plotted in Step 11, calculate an average time constant $\tau_c$ for the tank with orifice for the whole set of data. Follow graphical procedure outlined in Figure 6 in text.

13. By interpolation from same graph, using $h = 0.368\ h_{max}$, find time constant.

14. Calculate fluid capacitance of container from Equation 4.

15. Using $\tau_c$ obtained in step 12 or 13 and fluid capacitance C in step 14, calculate fluid resistance for each orifice used in the experiments.

16. Explain how the laboratory results confirm the principles given in the Discussion section.

| DATA TABLE 1: FIRST RESISTANCE | | |
|---|---|---|
| HEIGHT OF WATER LEVEL ABOVE ORIFICE (cm) | TIME TO EMPTY (s) | TIME OF FALL OF LIQUID LEVEL (s) |
| _____ ($h_{max}$) | _____ ($t_{max}$) | 0 |
| 40 | | |
| 35 | | |
| 30 | | |
| 25 | | |
| 20 | | |
| 15 | | |
| 10 | | |
| 5 | | |
| 2 | | |

Diameter of orifice = _____ cm

Diameter of cylindrical tank = _____ cm

| DATA TABLE 2: INCREASED RESISTANCE | | |
|---|---|---|
| HEIGHT OF WATER LEVEL ABOVE ORIFICE (cm) | TIME TO EMPTY (s) | TIME OF FALL OF LIQUID LEVEL (s) |
| _____ ($h_{max}$) | _____ ($t_{max}$) | 0 |
| 40 | | |
| 35 | | |
| 30 | | |
| 25 | | |
| 20 | | |
| 15 | | |
| 10 | | |
| 5 | | |
| 2 | | |

## SELF CHECK

1. Define "fluid capacitance" for a cylindrical storage tank.

2. A certain cylindrical water-storage tank has a cross-sectional area of 300 $m^2$ and an orifice resistance of $3 \times 10^7$ pa/($m^3$/s). Calculate the time constant of the tank and outlet system.

3. In a tank 10-m tall, the initial fluid level is 8 m above the outlet. The time constant for the system is 20 minutes. What is the level of the fluid above the outlet twenty-six minutes after the outlet is opened?

4. The time constant for a long, cylindrical, chemistry-laboratory buret is 3 seconds. What is the time required to empty the fluid contents out of the bottom outlet?

5. A storage tank of water of 15 m initial depth above the outlet and 75 $m^2$ cross-sectional area is drained through an outlet at the bottom. After 3 minutes, the level of the surface has dropped to 11 m. Calculate the following:

   a. The time constant of the tank-outlet system.
   b. The capacitance of the tank in $m^3$/pa.
   c. The outlet resistance in pa/($m^3$/s).
   d. The time required for the tank to drain.
   e. The volume flow rate at a fluid level 4 m above the outlet.

## REFERENCES


Dierauf, Edward J., Jr., and Court, James E. Unified Concepts in Applied Physics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1979.

Fribance, Austin E. Measurements Manual for Instrumentation and Applied Physics. Cambridge, MA: Hickok Teaching Systems, Inc., 1968.

Henke, Russell W. Introduction to Fluid Mechanics. Reading, MA: Addison-Wesley Publishing Company, 1966.

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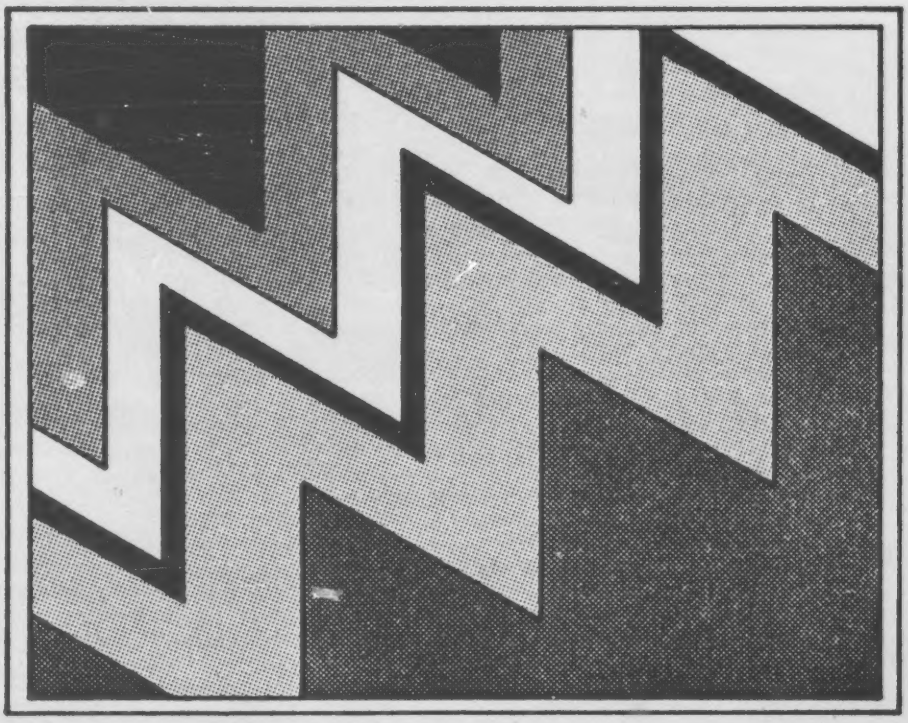
UNIFIED TECHNICAL CONCEPTS

MODULE 12 E 1

R C CIRCUITS



# ELECTRICAL APPLICATION OF TIME CONSTANTS

Technical Education Research Center - Southwest

TERC - SW

## INTRODUCTION

In electronic circuits, the necessity often arises to provide some kind of timing signal to synchronize other events. One of the most common sources of such a signal is a resistance-capacitance (RC) circuit. This circuit generates regularly-spaced pulses by charging and discharging a capacitor through a resistor.

This module examines the characteristics of RC timing circuits, including the time constant. In the laboratory, the student will construct and test an RC timing circuit.

## PREREQUISITES

Student should have completed Concept Module 12-0, "Time Constants."

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the "time constant" of an RC circuit.
- Calculate voltage on the capacitor of an RC circuit as a function of time, given "R," "C" and "V."
- Calculate time required for capacitor of an RC circuit to reach a given voltage.
- Given the appropriate equipment, construct an RC timing circuit; and measure its timing characteristics.

## DISCUSSION

Figure 1 shows an RC circuit. Assume that initial voltage on the capacitor is zero. When the switch is closed in the upper position, a current will flow through $R_1$ to charge the capacitor. Figure 2 shows voltage on the capacitor and voltage on resistor $R_1$ as functions of time. As voltage on capacitor increases, voltage on resistor decreases. Thus, less charge flows through the resistor to continue charging the capacitor. Voltage on the capacitor at any time is given by Equation 1.

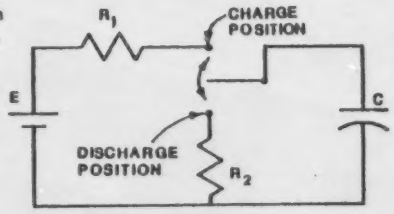



FIG. 1

$$V_c = V_T\ (1-e^{-t/\tau}) \qquad (1)$$

where

$V_c$ = instantaneous, charging voltage on capacitor.
$V_T$ = battery or applied voltage.
e = the natural base, approx. 2.718.
t = time in seconds.
$\tau$ = time constant (given by Equation 2).

The time constant $\tau$ is given by Equation 2.

$$\tau = RC \qquad (2)$$

where

R = resistance, in ohms, through which the capacitor charges.
C = capacitance, in farads, of the capacitor.

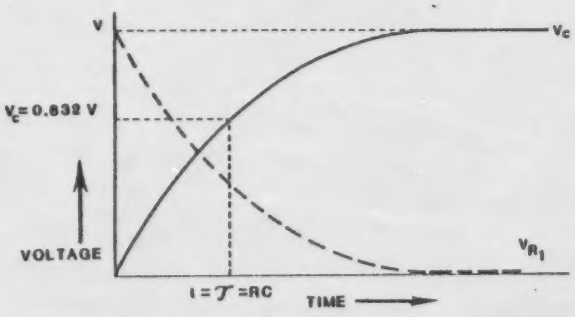



FIG. 2

The time constant is time required for capacitor to reach (1 - 1/e) or 0.632 of the battery voltage. This point on the $V_c$ curve is shown in Figure 2.

If the capacitor has been charged to voltage V, when switch is then moved to downward position, the capacitor will discharge through resistor $R_2$. Figure 3 shows capacitor voltage as a function of time for this case. Equation 3 gives this discharging voltage.

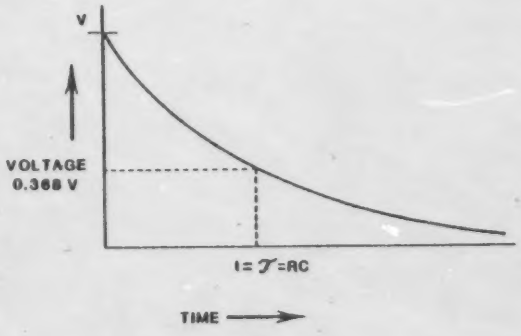



FIG. 3

Discharging: $V_c = e^{-t/\tau}$ (3)

In this case, the time constant $\tau = RC$ is time required for capacitor voltage to reach 1/e or 0.368 of original voltage.

The following examples (A, B, and C) show use of these equations in solving typical problems. Refer to Figure 1 for all examples; and use $C = 1\ \mu f$, $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$, $V_T = 100\ V$.

**EXAMPLE A: TIME CONSTANT CALCULATIONS**

Given: Figure 1 and $C = 1\ \mu f$, $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$, $V_T = 100\ V$.

Find: Time constants for charging and discharging circuits.

Solution: Charging: $\tau = RC$

$= (1000\ \Omega)(10^{-6}\ f)$

$\tau = 10^{-3}\ s$

Discharging: $\tau = RC$

$= (2000\ \Omega)(10^{-6}\ f)$

$\tau = 2 \times 10^{-3}\ s$

**EXAMPLE B: CALCULATION OF CAPACITOR VOLTAGE IN CHARGE MODE**

Given: Initial capacitor voltage is zero, and switch is closed upward.

Find: Voltage on capacitor after 1 ms.

Solution: $V_c = V_T\ (1 - e^{-t/\tau})$

$= (100\ V)[1 - e^{-\frac{(0.001)}{(0.001)}}]$

$= 100\ V\ [1 - e^{-1}]$

$= 100\ V\ [1 - 0.368]$

$V_c = 63.2\ V$

(Note: 1 ms is time constant for charging circuit.)

**EXAMPLE C: CALCULATION OF CAPACITOR VOLTAGE IN DISCHARGE MODE**

**Given:** Initial capacitor voltage is 100 V, and switch is closed downward.

**Find:** Capacitor voltage after 6 ms.

**Solution:**

$$V_c = V_T e^{-t/\tau}$$

$$= (100\ V)\ e^{-0.006/0.002}$$

$$= (100\ V)\ e^{-3} \qquad \text{NOTE: } e^{-3} = 0.0498$$

$$= (100\ V)(0.0498)$$

$$V_c = 4.98\ V$$

In practical timing circuits, some type of sensing device is attached to the capacitor. This device delivers an output signal when capacitor voltage reaches some preset value. When such circuits are designed, the time required for the capacitor to reach a given voltage must be determined. This time value is calculated from Equations 4 and 5.

$$\text{Charging:} \qquad t = -\tau \ln \left(1 - \frac{V_c}{V_T}\right) \qquad (4)$$

$$\text{Discharging:} \qquad t = -\tau \ln \left(\frac{V_c}{V_T}\right) \qquad (5)$$

where

t = time required for capacitor to reach a given voltage.

Examples D and E show use of Equations 4 and 5 in solving problems.

**EXAMPLE D: CALCULATION OF TIME TO REACH A GIVEN VOLTAGE IN CHARGE MODE**

**Given:** From Figure 1, capacitor fully discharged and switch closed upward. ($C = 1\ \mu f$, $R_1 = 1000\ \Omega$, $R_2 = 2000\ \Omega$, $V_T = 100\ V$.)

**Find:** Time before capacitor voltage is 67 V.

**Solution:**

$$t = -\tau \ln \left(1 - \frac{V_c}{V_T}\right), \text{ where } \tau = R_1 C$$

$$= -(1000\ \Omega)(10^{-6}\ f) \ln \left(1 - \frac{67\ V}{100\ V}\right)$$

$= -(10^{-3}\ s)\ \ln\ (0.33)$

$\therefore \tau = 1.11 \times 10^{-3}\ s = 1.11\ ms$

(Compare with Example B.)

**EXAMPLE E: CALCULATION OF TIME TO REACH A GIVEN VOLTAGE IN DISCHARGE MODE**

Given: Capacitor fully charged to 100 V and switch closed downward.

Find: Time before capacitor voltage reaches 33 V.

Solution: $t = -\tau \ln \left(\frac{V_C}{V_T}\right)$, where $\tau = R_2C = 2000\,\Omega \times 10^{-6}\ f = 2\ ms$

$= -(2000\ \Omega)(10^{-6}\ f)\ \ln \left(\frac{33}{100}\right)$

$= -(2 \times 10^{-3})\ \ln\ (0.33)$

$\therefore t = 2.2 \times 10^{-3}\ s$ (Compare with time constant of this discharge circuit.)

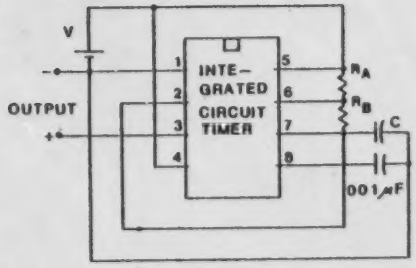



FIG. 4

One of the most common sensing devices used with RC circuits is the 555 integrated circuit timer. Figure 4 shows this device in a clock circuit. The applied voltage V is between 5 and 15 volts. Capacitor C charges through $R_A$ and $R_B$ in series until a value of 2/3 V is reached. The timer then switches to discharge C through $R_B$ alone until the capacitor voltage is 1/3 V. At this time, the charging phase is repeated. During charging the output voltage is V, and during discharging the output voltage is zero. Figure 5 shows capacitor voltage and output voltage of this circuit. Note that output voltage varies from 0-10 V, while capacitor voltage goes from 3.33 V to 6.67 V during charge process and from 6.67 V to 3.33 V during discharge process. Also note that time for charge $t_1$ (Equation 6) is 47.8 μs, and time for dischage $t_2$ (Equation 7) is 20.8 μs. These values are based on $R_A$ = 3.9 kΩ, $R_B$ = 3.0 kΩ and C = 0.01 μf as given at the bottom of Figure 5.

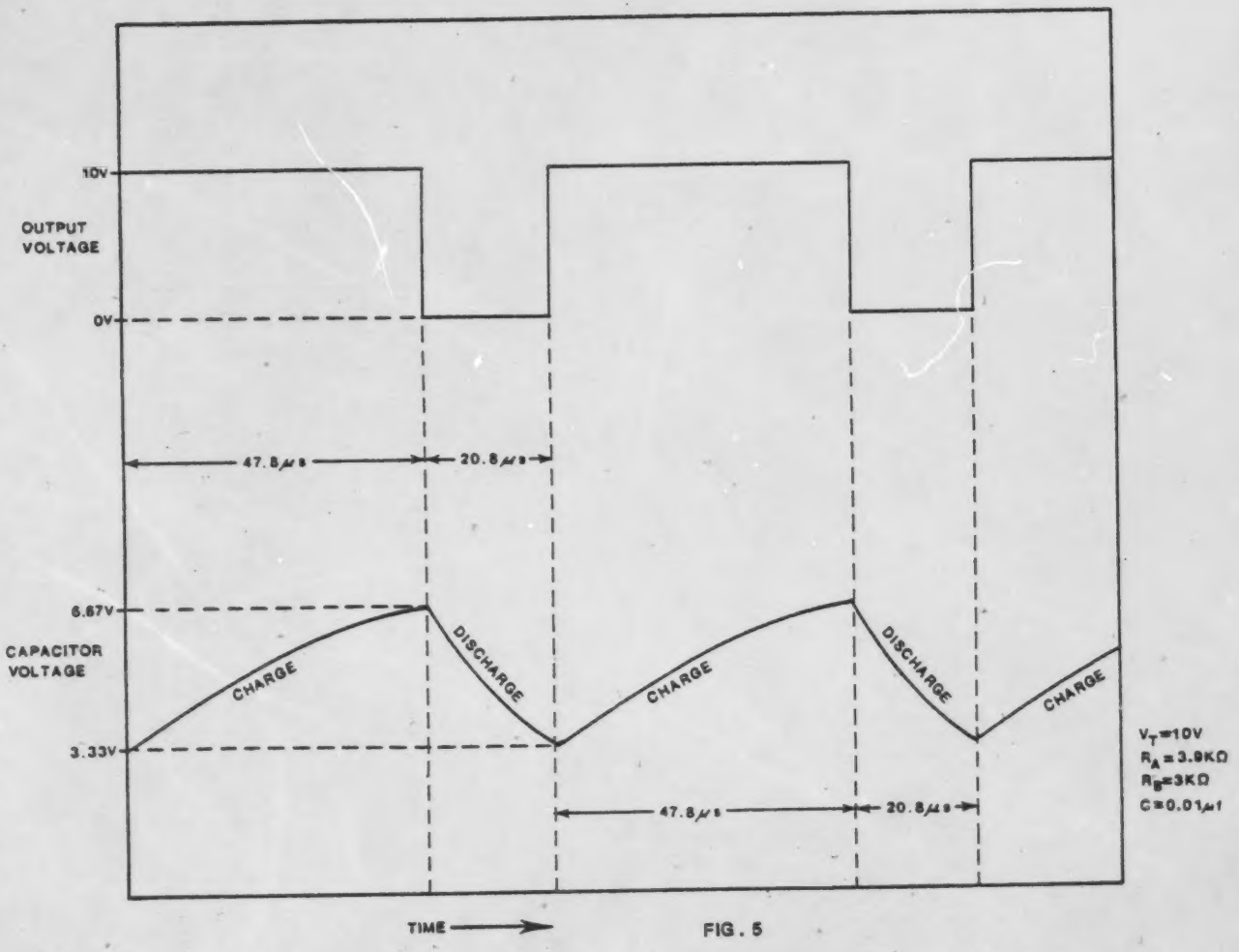

10V
OUTPUT VOLTAGE
0V
47.8μs
20.8μs
6.67V
CAPACITOR VOLTAGE
3.33V
CHARGE
DISCHARGE
CHARGE
DISCHARGE
CHARGE
47.8μs
20.8μs
TIME
$V_T$=10V
$R_A$=3.9KΩ
$R_B$=3KΩ
C=0.01μf

FIG. 5

For the particular circuit in Figure 5, Equations 4 and 5 become Equations 6 and 7 as follows:

Charge:

$$t_1 = 0.693\ (R_A + R_B)\ C \qquad (6)$$

where

$t_1$ = charge time.
$(R_A + R_B)\ C$ = time constant $\tau$ during charging process.

Discharge:

$$t_2 = 0.693\ (R_B)\ C \qquad (7)$$

where

$t_2$ = discharge time.
$(R_B)\ C$ = time constant $\tau$ during discharging process.

The total period then is the following:

$$t_T = t_1 + t_2 = 0.693\ (R_A + 2\ R_B)\ C$$

where

$t_T$ = total time period.

Frequency of oscillation is then:

$$f = \frac{1}{t_T} = \frac{1.44}{(R_A + 2\ R_B)\ C}$$

where

f = frequency of oscillation.

The type of circuit described here is the heart of many electronic timing devices, including digital watches, clocks, and time-delay circuits. It is adequate for most timing applications, in which extreme accuracy and stability are not required.

## SUMMARY

The RC circuit is widely used as a source of timing signals for the synchronization of other events or processes. It generates regularly-spaced pulses as follows: A capacitor is charged and discharged through a resistor.

## TIMING CHARACTERISTICS OF AN RC CIRCUIT

### MATERIALS

555 timer (integrated circuit).
10-V d.c. power supply.
6-kΩ resistor.
2-kΩ resistor.
0.01-μF capacitor.
0.22-μF capacitor.
Oscilloscope.
Clip leads.
Terminal board.

### PROCEDURE

1. Connect circuit as shown in Figure 4 with $R_A$ = 6 kΩ, $R_B$ = 2 kΩ, C = 0.22 μF. Do not turn on power supply.

2. Complete portion of Data Table entitled Predicted Results.

3. Connect oscilloscope across capacitor C. Set voltage sensitivity for 1 V/cm and time base for 0.5 ms/cm. Turn on power supply and oscilloscope and observe trace. Sketch oscilloscope trace in Data Table.

4. Connect oscilloscope to timer output, change voltage sensitivity to 5 V/cm, and observe and sketch trace.

5. Measure and record charging and discharging times of capacitor and period of output waveform.

6. Explain the timing features of this circuit.

| DATA TABLE |
| --- |
| PREDICTED RESULTS. |
| Charging time: $t_1 = 0.693\ (R_A + R_B)\ C$ |
| $= 0.693\ (____ + ____)(____)$ |

$t_1$ = _____ s

Discharging time: $t_2 = 0.693\ (R_B)\ C$

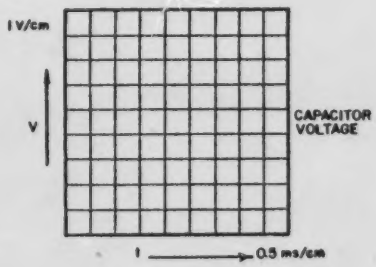
$= 0.693\ (_____)(_____)$

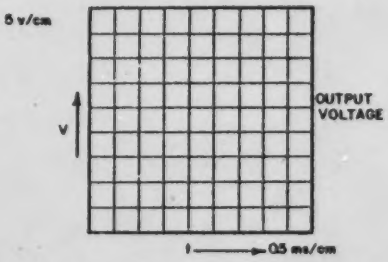
$t_2$ = _____ s

Period: $t_T = \dfrac{1.44}{(R_A + 2\ R_B)\ C} = \dfrac{1.44}{(_____ + 2\ _____)(_____)}$

$t_T$ = _____ s

EXPERIMENTAL RESULTS




Charging time: $t_1$ = _____ s

Discharging time: $t_2$ = _____ s

Period: $t_T$ = _____ s

## SELF CHECK

1. Define "time constant" of an RC circuit.

2. For an RC circuit, the following values are given: R = 10 kΩ, C = 0.44 μf, $V_T$ = 12 V. If capacitor begins charging from $V_C$ = 0, calculate its voltage after 5 ms.

3. After the above capacitor is fully charged to 12 V, it is discharged through a 2-kΩ resistor. Calculate capacitor voltage 1 ms into discharge phase.

4. Calculate time required for capacitor in problem 2 to reach 10 V.

5. Calculate time required for capacitor in problem to reach 5 V.

6. A 555 timer and a 0.01-μf capacitor are to be used in a timing circuit. Calculate $R_A$ and $R_B$ for a charging time of 0.75 ms and a discharging time of 0.25 ms.

## REFERENCES


Faber, Rodney B. Applied Electricity and Electronics for Technology. New York: John Wiley & Sons, 1978.

Jackson, Herbert W. Introduction to Electric Circuits. 4th ed. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1976.

Klaylon, Marvin H. Fundamental Electrical Technology. Reading, MA: Addison-Wesley Publishing Company, 1977.

Sears, Francis W.; Zemansky, Mark W.; and Young, Hugh D. University Physics. 5th ed. Reading, MA: Addison-Wesley Publishing Company, 1976.

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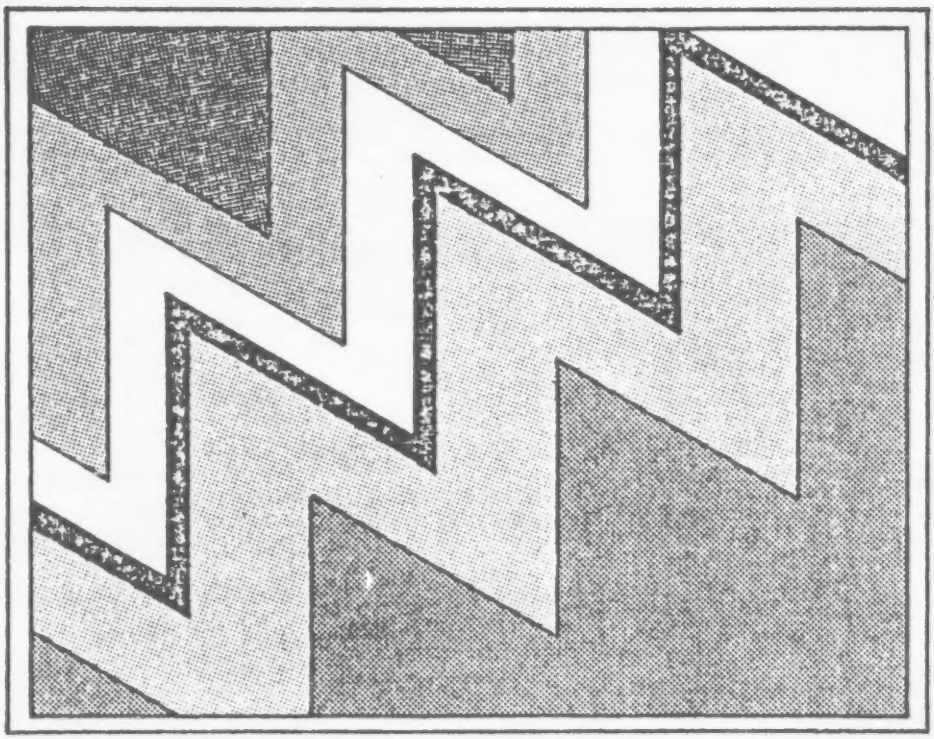
UNIFIED TECHNICAL CONCEPTS

MODULE 12-E-2

RLC CIRCUITS



# ELECTRICAL APPLICATION OF TIME CONSTANTS CONCEPT

Technical Education Research Center – Southwest

TERC – SW

## INTRODUCTION

In many electronic circuits, it is necessary to deliver electrical power in the form of pulses. For low-energy pulses, resistance-capacitance (RC) circuits may be used. But RC circuits have characteristics that make them undesirable for higher-energy pulses. In all high-energy pulsed circuits and in many low-energy ones, an inductor is added to shape the pulse.

In this module, we shall examine characteristics and applications of resistance-inductance-capacitance (RLC) circuits. The student will determine proper values for critically damped RLC circuits and will examine pulse shapes of overdamped, underdamped and critically damped RLC circuits in the laboratory.

## PREREQUISITES

The student should have completed Concept Module 12-0, "Time Constants," and Application Module 12E1, "RC Circuits," and should have ability to solve simple algebraic equations and make measurements of voltage and time with an oscilloscope.

## OBJECTIVES

To complete this module, the student should be able to:

- Draw pulse shape from (a) an overdamped RLC circuit (and explain its disadvantages), (b) an underdamped RLC circuit (and explain its disadvantages), (c) a critically damped RLC circuit.
- Given capacitance and resistance of an RLC circuit, calculate inductance necessary for critical damping and pulse duration at one third of maximum height.
- Given necessary equipment, construct RLC circuits and observe pulses from overdamped, underdamped and critically damped circuits.

## DISCUSSION

Figure 1 shows the schematic of a typical RLC circuit. In this circuit, the capacitor stores energy that will be delivered to the load resistor R in the form of a pulse. The inductor L is a current limiting device that is used to control shape of the pulse.

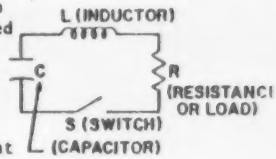



Fig. 1

If inductance is very small, the circuit is said to be overdamped. In this case, circuit performance approximates that of an RC circuit discussed in Module 12E1. Figure 2 shows current through the resistor as a function of time after the switch is closed.

When the switch is closed, current rises rapidly to a peak value of:

$$I = \frac{V}{R}$$

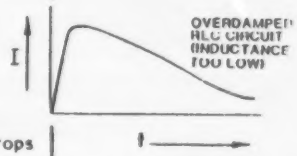



Fig. 2

where V is initial voltage of the capacitor. Current then drops off exponentially as in an RC circuit. The major difficulty with such pulses is the sharp initial rise in current. This rapid change in current produces both thermal and mechanical shocks in load and will greatly decrease useful life of the load resistor. In flashlamp circuits, this sharp current peak is often responsible for lamp burn out and reduces number of shots available from a lamp even if it does not prematurely burn out.

Figure 3 shows the pulse produced by an underdamped RLC circuit. In this case, inductance is larger than the optimum value. Since the inductor resists changes in current, it smooths out the leading edge of the current pulse. When capacitor voltage reaches zero, the inductor tries to maintain current flow at a high level. This causes a reverse charge on the capacitor. At some point, current begins to drop. When it reaches zero, the capacitor has a large reverse charge. This charge causes a current through the load resistance in the opposite direction from the original current. Thus, the circuit oscillates or "rings" until all original energy stored in the capacitor is dissipated. The ringing of such a circuit results in inefficiency of power delivery to the load resistor and may damage energy storage capacitors by rapidly reversing their polarity.

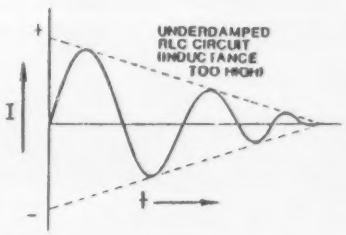



Fig. 3

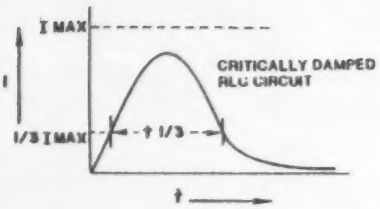



Fig. 4

Figure 4 shows the pulse from a critically damped RLC circuit. In this circuit, value of inductance has been chosen to be large enough to prevent the sharp initial current rise but small enough to prevent "ringing" of the circuit. This delivers energy to the load in the most efficient manner without subjecting either load or energy storage capacitor to unnecessary damaging conditions.

In critically damped RLC circuits, pulse duration is usually measured as full width of the pulse at 1/e of the maximum current. Since 1/e = 0.368, an approximation of 0.333 or 1/3 of maximum height is often used (Figure 4). Pulse duration of such a circuit is given by Equation 1.

$$\text{Pulse duration at } \tfrac{1}{3} \text{ maximum} = \text{resistance of load} \times \text{capacitance.}$$

$$\text{In symbols: } \Delta t_{1/3} = RC. \qquad (1)$$

For critical damping of an RLC circuit, inductance is given by Equation 2.

$$\text{Inductance} = \frac{\text{square of resistance} \times \text{capacitance}}{4}.$$

$$\text{In symbols: } L = \frac{R^2C}{4}. \qquad (2)$$

Examples A and B demonstrate use of Equations 1 and 2 in solving problems.

Example B represents the most common type of problem encountered. Load resistance and desired pulse duration are usually known. From these values, necessary capacitance and inductance may be determined.

Energy delivered to the load is energy stored in the capacitor as given in Equation 3.

**EXAMPLE A: TYPICAL RLC CIRCUIT PROBLEM**

Given: An RLC circuit designed to use a 0.4-Ω and a 1000-uf energy storage capacitor.

Find: (a) Pulse duration and (b) inductance necessary for critical damping.

Solution:
(From Equation 1)

(a) $\Delta t_{1/3} = RC$

$= (0.4\ \Omega)(1000 \times 10^{-6}\ f)$

$= 0.4 \times 10^{-3}\ s$

$\therefore \Delta t_{1/3} = 0.4\ ms.$

(From Equation 2)

(b) $L = \frac{R^2C}{4}$

$= (0.25)(0.4\ \Omega)^2(1000 \times 10^{-6}\ f)$

$= 4 \times 10^{-5}\ H$

$L = 4.0\ \mu H = 40$ microhenries.

**EXAMPLE B: TYPICAL RLC CIRCUIT PROBLEM**

Given: RLC circuit designed to deliver a 1-ms pulse to a 1-Ω load.

Find: (a) Capacitance necessary and (b) inductance necessary for critical damping.

Solution:

(a) $\Delta t_{1/3} = RC$

$C = \frac{\Delta t_{1/3}}{R}$

$= \frac{10^{-3}\ s}{1\ \Omega}$

$= 10^{-3}\ f$

$\therefore C = 1000\ uf = 1000$ microfarads.

(b) $L = \frac{R^2C}{4}$

$= (0.25)(1\ \Omega)^2(1000 \times 10^{-6}\ f)$

$= 2.5 \times 10^{-4}\ H$

$\therefore L = 250\ \mu H = 250$ microhenries.

Energy = $\frac{1}{2}$ x capacitance x square of voltage.

In symbols: $E = \frac{1}{2} CV^2$. (3)

Example C shows use of this equation in solving a problem.

**EXAMPLE C: CALCULATION OF ENERGY STORED IN A CAPACITOR**

Given: Energy delivered to the load in Example A must be 500 J.

Find: Voltage to which capacitor must be charged.

Solution:
(From Equation 3)

$$E = \frac{CV^2}{2}$$

From which,

$$V^2 = \frac{2E}{C}$$

$$V = \sqrt{\frac{2E}{C}}$$

$E = 500$ J

$C = 1000$ μf.

Substituting,

$$V = \sqrt{\frac{2 \times 500}{1000 \times 10^{-6}}}$$

$$V = 10^6$$

$\therefore V = 1000$ volts.

Capacitor must be charged to 1000 volts!

RLC circuits are employed when high-energy pulses must be delivered to a load. Common applications involve generation of high-power radar pulses or pulsed magnetic fields for research. A common application involves production of optical pulses from flashlamps. This includes photographic strobes and optically

pumped pulsed laser power supplies.

A simplified flashlamp power supply is shown in Figure 5. Resistance of the circuit is lamp resistance after breakdown and is typically between 0.3 Ω and 1.0 Ω. Voltage on the capacitor is sufficient to maintain a current through the lamp but is not high enough to cause electrical breakdown of the gas within the envelope. A trigger pulse of several thousand volts is supplied by a trigger transformer. When this pulse is applied to an external electrode on the lamp, the gas is ionized and lamp resistance drops from a previously very high value to a much lower operating level.

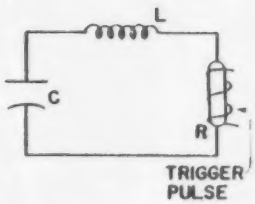



Fig. 5

## MATERIALS

Oscilloscope
VOM.
Photoflash, flashlamp (National Camera F4407 or equivalent).
Trigger transformer (Archer 272-1146 or equivalent).
550-μF, 400-V capacitor.
0.022-μF, 400-V capacitor.
d.c. power supply, 300-V capability, adjustable.
Push-button switch, mounted.
Clip leads.
50-μH inductor.
200-μH inductor.
50-kΩ, 2-W resistor.
2-MΩ, 1-W resistor.
100-kΩ, 1-W resistor.
0.1-Ω, 2-W resistor or resistance wire to make 0.1-Ω resistor.

## PROCEDURE

CAUTION: Dangerous high voltages are present in the circuit used in this experiment. Do not touch components while power is on. Turn off power supply and discharge all capacitors before changing wiring connections.

1. Use clip leads to connect circuit as shown in Figure 6. Turn on power supply; adjust control to obtain a voltage of 300 V as measured with VOM.
2. Place a square of black paper over lamp to reduce amount of room light falling on lamp. Do not look directly at lamp when it is fired.
3. Press push button. If lamp does not fire, have lab instructor check circuit. Allow 15 seconds between shots for lamp to cool.
4. Turn on oscilloscope; set it for 10 V/cm and 0.2 ms/cm. Triggering should be set to give a single sweep when lamp is fired. If you do not understand how to do this, ask instructor for assistance.

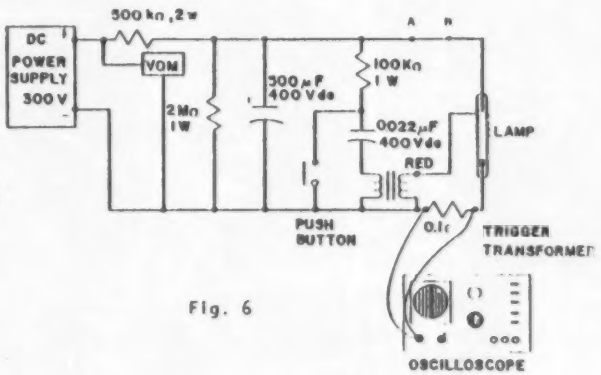



Fig. 6

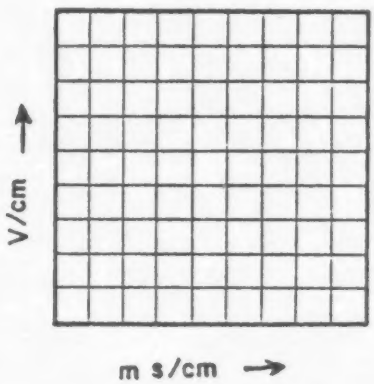
5. Fire lamp several times and observe pulse. Make a sketch of pulse and complete part A of data table.
6. Connect 200-μH inductor between points A and B of Figure 6; repeat above steps. Complete part B of data table. Be sure to note "ringing" frequency.
7. Replace 200-μH inductor with 50-μH inductor; repeat again. Complete part C of data table.

NOTE: The 0.1-Ω resistor in the circuit is called a "current-viewing" resistor and is present for measurement only. It is not normally included in a flashlamp circuit.

**DATA TABLE**

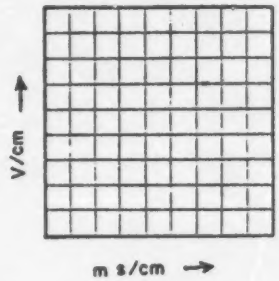
Part A: Overdamped RLC circuit

Sketch pulse produced with zero inductance between A and B. Include pulse duration.



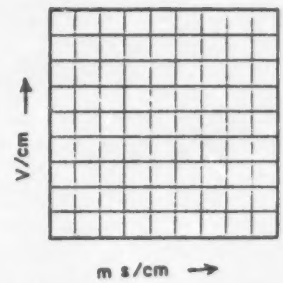
Part B: Underdamped RLC circuit

Sketch pulse produced with an inductance of 200 μH between A and B. Include ringing frequency.




Part C: Critically damped RLC circuit

Sketch pulse produced with an inductance of 50 μH between A and B. Include pulse duration.



## SELF CHECK

1. Draw pulse shape from an overdamped RLC circuit; explain its disadvantages. Why is circuit overdamped?
2. Draw pulse shape from an underdamped RLC circuit; explain its disadvantages. Why is circuit underdamped?
3. Draw pulse shape from a critically damped RLC circuit.
4. 600 J of energy is to be delivered to a 0.5-Ω load with a pulse duration 0.5 ms. Find:

   Capacitance necessary.
   Inductance necessary for critical damping.
   Voltage to which energy storage capacitor must be charged.

5. Use pulse duration measured for the critically damped circuit in experiment and capacitance of the circuit; calculate resistance of the load.

## REFERENCES

Technical Education Research Center - SW, ed. "Pulsed Laser Flashlamps and Power Supplies." Module 3-1, Laser and Electro-Optic Components Series, developed pursuant to USOE Grant #OEG-0-8-080491-4701 (085).

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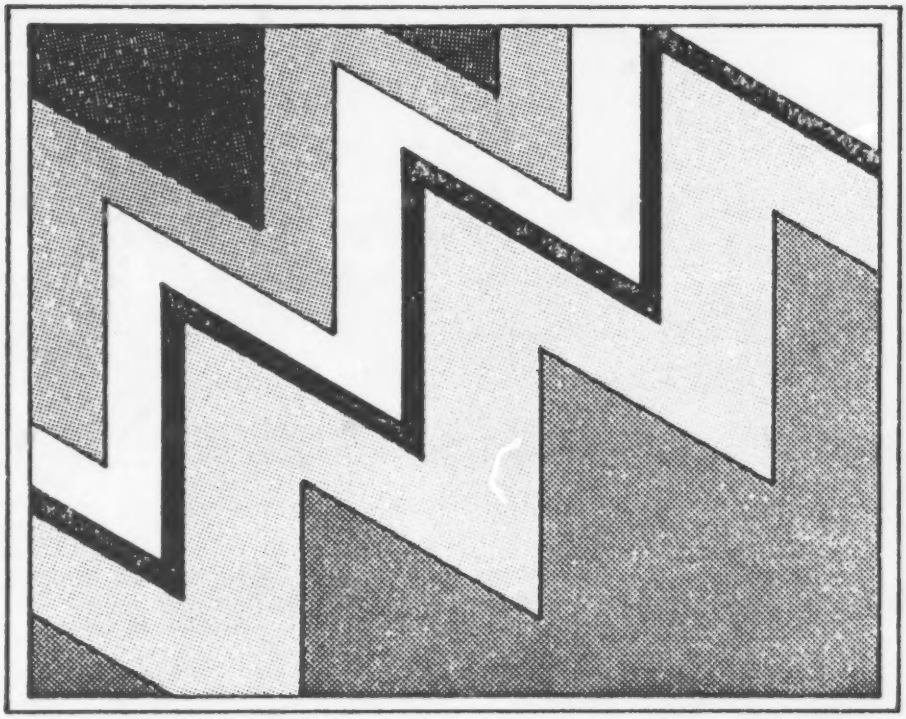
UNIFIED TECHNICAL CONCEPTS

MODULE 12 E 3

R L C CIRCUITS



# ELECTRICAL APPLICATION OF TIME CONSTANTS

Technical Education Research Center – Southwest

TERC – SW

## INTRODUCTION

In many electronic circuits, it is necessary to deliver electrical power in the form of pulses. For low-energy pulses, resistance-capacitance (RC) circuits may be used. But RC circuits have characteristics that make them undesirable for higher-energy pulses. In all high-energy pulsed circuits and in many low-energy ones, an inductor is added to shape the pulse.

In this module, we shall examine characteristics and applications of resistance-inductance-capacitance (RLC) circuits. The student will determine proper values for critically damped RLC circuits and will examine pulse shapes of overdamped, underdamped and critically damped RLC circuits in the laboratory.

## PREREQUISITES

The student should have completed Concept Module 12-0, "Time Constants," and Application Module 12E1, "RC Circuits."

## OBJECTIVES

To complete this module, the student should be able to:

- Draw pulse shape from (a) an overdamped RLC circuit (and explain its disadvantages), (b) an underdamped RLC circuit (and explain its disadvantages), (c) a critically damped RLC circuit.
- Given capacitance and resistance of an RLC circuit, calculate inductance necessary for critical damping and pulse duration at one third of maximum height.
- Given necessary equipment, construct RLC circuits and observe pulses from overdamped, underdamped and critically damped circuits.

## DISCUSSION

Figure 1 shows the schematic of a typical RLC circuit. In this circuit, the capacitor stores energy that will be delivered to the load resistor R in the form of a pulse. The inductor L is a current limiting device that is used to control shape of the pulse.

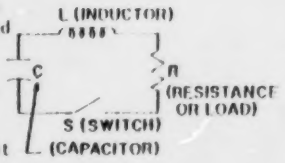



Fig. 1

If inductance is very small, the circuit is said to be overdamped. In this case, circuit performance approximates that of an RC circuit discussed in Module 12E1. Figure 2 shows current through the resistor as a function of time after the switch is closed.

When the switch is closed, current rises rapidly to a peak value of:

$$I = \frac{V}{R}$$

Fig. 2

where V is initial voltage of the capacitor. Current then drops off exponentially as in an RC circuit. The major difficulty with such pulses is the sharp initial rise in current. This rapid change in current produces both thermal and mechanical shocks in load and will greatly decrease useful life of the load resistor. In flashlamp circuits, this sharp current peak is often responsible for lamp burn out and reduces number of shots available from a lamp even if it does not prematurely burn out.

Figure 3 shows the pulse produced by an underdamped RLC circuit. In this case, inductance is larger than the optimum value. Since the inductor resists changes in current, it smooths out the leading edge of the current pulse. When capacitor voltage reaches zero, the inductor tries to maintain current flow at a high level. This causes a reverse charge on the capacitor. At some point, current begins to drop. When it reaches zero, the capacitor has a large reverse charge. This charge causes a current through the load resistance in the opposite direction from the original current. Thus, the circuit oscillates or "rings" until all original energy stored in the capacitor is dissipated. The ringing of such a circuit results in inefficiency of power delivery to the load resistor and may damage energy storage capacitors by rapidly reversing their polarity.

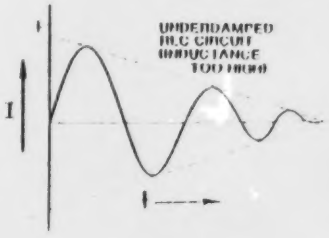



Fig. 3

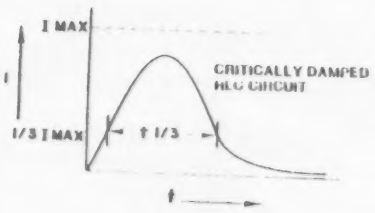



Fig. 4

Figure 4 shows the pulse from a critically damped RLC circuit. In this circuit, value of inductance has been chosen to be large enough to prevent the sharp initial current rise but small enough to prevent "ringing" of the circuit. This delivers energy to the load in the most efficient manner without subjecting either load or energy storage capacitor to unnecessary damaging conditions.

In critically damped RLC circuits, pulse duration is usually measured as full width of the pulse at 1/e of the maximum current. Since 1/e = 0.368, an approximation of 0.333 or 1/3 of maximum height is often used (Figure 4). Pulse duration of such a circuit is given by Equation 1.

$$\text{Pulse duration at } \frac{1}{3} \text{ maximum} = \text{resistance of load} \times \text{capacitance.}$$

$$\text{In symbols: } \Delta t_{1/3} = RC. \qquad (1)$$

For critical damping of an RLC circuit, inductance is given by Equation 2.

$$\text{Inductance} = \frac{\text{square of resistance} \times \text{capacitance}}{4}.$$

$$\text{In symbols: } L = \frac{R^2C}{4}. \qquad (2)$$

Examples A and B demonstrate use of Equations 1 and 2 in solving problems.

Example B represents the most common type of problem encountered. Load resistance and desired pulse duration are usually known. From these values, necessary capacitance and inductance may be determined.

Energy delivered to the load is energy stored in the capacitor as given in Equation 3.

**EXAMPLE A: TYPICAL RLC CIRCUIT PROBLEM**

Given: An RLC circuit designed to use a 0.4-Ω and a 1000-μf energy storage capacitor.

Find: (a) Pulse duration and (b) inductance necessary for critical damping.

Solution:

(From Equation 1)

(a) $\Delta t_{1/3} = RC$

$= (0.4\ \Omega)(1000 \times 10^{-6}\ f)$

$= 0.4 \times 10^{-3}\ s$

$\therefore \Delta t_{1/3} = 0.4\ ms.$

(From Equation 2)

(b) $L = \frac{R^2C}{4}$

$= (0.25)(0.4\ \Omega)^2(1000 \times 10^{-6}\ f)$

$= 4 \times 10^{-5}\ H$

$L = 4.0\ \mu H = 40$ microhenries.

**EXAMPLE B: TYPICAL RLC CIRCUIT PROBLEM**

Given: RLC circuit designed to deliver a 1-ms pulse to a 1-Ω load.

Find: (a) Capacitance necessary and (b) inductance necessary for critical damping.

Solution:

(a) $\Delta t_{1/3} = RC$

$C = \frac{\Delta t_{1/3}}{R}$

$= \frac{10^{-3}\ s}{1\ \Omega}$

$= 10^{-3}\ f$

$\therefore C = 1000\ \mu f = 1000$ microfarads.

(b) $L = \frac{R^2C}{4}$

$= (0.25)(1\ \Omega)^2(1000 \times 10^{-6}\ f)$

$= 2.5 \times 10^{-4}\ H$

$\therefore L = 250\ \mu H = 250$ microhenries.

$$\text{Energy} = \frac{1}{2} \times \text{capacitance} \times \text{square of voltage.}$$

In symbols: $E = \frac{1}{2} CV^2$. (3)

Example C shows use of this equation in solving a problem.

**EXAMPLE C: CALCULATION OF ENERGY STORED IN A CAPACITOR**

Given: Energy delivered to the load in Example A must be 500 J.

Find: Voltage to which capacitor must be charged.

Solution:
(From Equation 3)

$$E = \frac{CV^2}{2}$$

From which,

$$V^2 = \frac{2E}{C}$$

$$V = \sqrt{\frac{2E}{C}}$$

$E = 500$ J

$C = 1000$ μf.

Substituting,

$$V = \sqrt{\frac{2 \times 500}{1000 \times 10^{-6}}}$$

$$V = 10^6$$

$\therefore V = 1000$ volts.

Capacitor must be charged to 1000 volts!

RLC circuits are employed when high-energy pulses must be delivered to a load. Common applications involve generation of high-power radar pulses or pulsed magnetic fields for research. A common application involves production of optical pulses from flashlamps. This includes photographic strobes and optically

pumped pulsed laser power supplies.

A simplified flashlamp power supply is shown in Figure 5. Resistance of the circuit is lamp resistance after breakdown and is typically between 0.3 Ω and 1.0 Ω. Voltage on the capacitor is sufficient to maintain a current through the lamp but is not high enough to cause electrical breakdown of the gas within the envelope. A trigger pulse of several thousand volts is supplied by a trigger transformer. When this pulse is applied to an external electrode on the lamp, the gas is ionized and lamp resistance drops from a previously very high value to a much lower operating level.

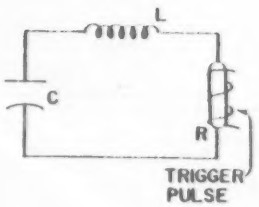



Fig. 5

## MATERIALS

Oscilloscope
VOM.
Photoflash, flashlamp (National Camera 54407 or equivalent).
Trigger transformer (Archer 272-1146 or equivalent).
550-μF, 400-V capacitor.
0.022-μF, 400-V capacitor.
d.c. power supply, 300-V capability, adjustable.
Push-button switch, mounted.
Clip leads.
50-μH inductor.
200-μH inductor.
50-kΩ, 2-W resistor.
2-MΩ, 1-W resistor.
100-kΩ, 1-W resistor.
0.1-Ω, 2-W resistor or resistance wire to make 0.1-Ω resistor.

## PROCEDURE

CAUTION: Dangerous high voltages are present in the circuit used in this experiment. Do not touch components while power is on. Turn off power supply and discharge all capacitors before changing wiring connections.

1. Use clip leads to connect circuit as shown in Figure 6. Turn on power supply; adjust control to obtain a voltage of 300 V as measured with VOM.
2. Place a square of black paper over lamp to reduce amount of room light falling on lamp. Do not look directly at lamp when it is fired.
3. Press push button. If lamp does not fire, have lab instructor check circuit. Allow 15 seconds between shots for lamp to cool.
4. Turn on oscilloscope; set it for 10 V/cm and 0.2 ms/cm. Triggering should be set to give a single sweep when lamp is fired. If you do not understand how to do this, ask instructor for assistance.

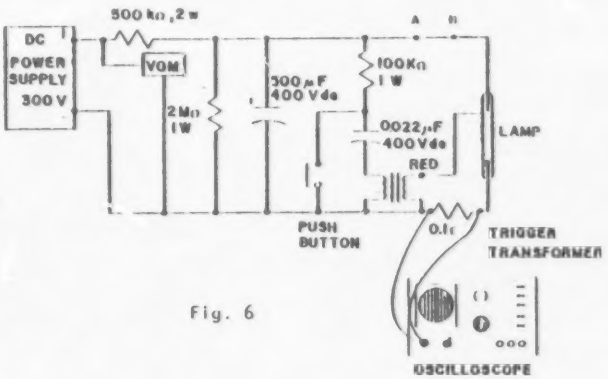



Fig. 6

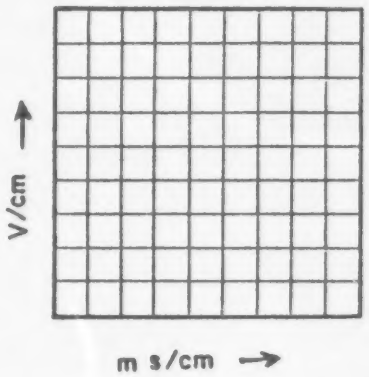
5. Fire lamp several times and observe pulse. Make a sketch of pulse and complete part A of data table.
6. Connect 200-μH inductor between points A and B of Figure 6; repeat above steps. Complete part B of data table. Be sure to note "ringing" frequency.
7. Replace 200-μH inductor with 50-μH inductor; repeat again. Complete part C of data table.

NOTE: The 0.1-Ω resistor in the circuit is called a "current-viewing" resistor and is present for measurement only. It is not normally included in a flashlamp circuit.

**DATA TABLE**

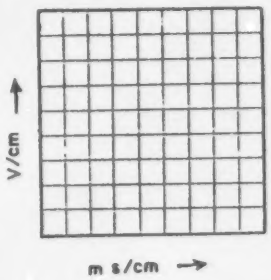
Part A: Overdamped RLC circuit

Sketch pulse produced with zero inductance between A and B. Include pulse duration.



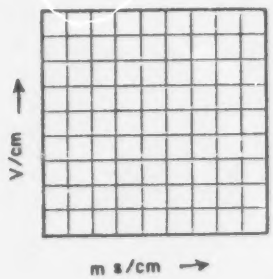
Part B: Underdamped RLC circuit

Sketch pulse produced with an inductance of 200 μH between A and B. Include ringing frequency.




Part C: Critically damped RLC circuit

Sketch pulse produced with an inductance of 50 μH between A and B. Include pulse duration.



## SELF CHECK

1. Draw pulse shape from an overdamped RLC circuit; explain its disadvantages. Why is circuit overdamped?
2. Draw pulse shape from an underdamped RLC circuit; explain its disadvantages. Why is circuit underdamped?
3. Draw pulse shape from a critically damped RLC circuit.
4. 600 J of energy is to be delivered to a 0.5-Ω load with a pulse duration 0.5 ms. Find:

   Capacitance necessary.
   Inductance necessary for critical damping.
   Voltage to which energy storage capacitor must be charged.

5. Use pulse duration measured for the critically damped circuit in experiment and capacitance of the circuit; calculate resistance of the load.

## REFERENCES

Technical Education Research Center - SW, ed. "Pulsed Laser Flashlamps and Power Supplies." Module 3-1, Laser and Electro-Optic Components Series, developed pursuant to USOE Grant #OEG-0-8-080491-4701 (085).

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UNIFIED TECHNICAL CONCEPTS

MODULE 12 T 1

# RESPONSE TIME OF THERMOCOUPLES

## THERMAL APPLICATION OF TIME CONSTANTS

Technical Education Research Center – Southwest

TERC – SW

## INTRODUCTION

A thermocouple consists of a pair of wires of dissimilar metals joined together at one end, with opposite ends connected to a voltage-measuring device. When temperature of the junction between the wires changes relative to the other ends, a voltage is generated. Voltage varies in relation to temperature change and can be used to monitor temperature.

Time is required for thermal energy (heat) to move through materials by thermal conduction. If one end of a metal spoon is placed in hot water, after some length of time, the other end will become warm. The rate of such heating effects can be discussed in terms of convenient time intervals, referred to as "time constants." In particular, a temperature-measuring device monitoring temperature of a certain sample takes time to respond to a change in the sample's temperature. Its response time is governed by the time required for thermal conduction of heat from the sample to the measuring device.

This module discusses heat in transient situations; that is, in situations in which temperature is changing rapidly. Under these conditions, response time of a temperature-measuring device becomes important. Response time of a thermocouple depends upon its size, its material, and its bonding to the material under study. Small, thin-film thermocouples, for example, have short response times (microseconds).

In the laboratory, the student will measure response times for a conventional thermocouple and a thin-film thermocouple and will use a thermocouple to measure thermal diffusivity of a sample of metal.

## PREREQUISITES

Student should have completed Concept Module 12-0, "Time Constants."

# OBJECTIVES

Upon completion of this module, the student should be able to:

- Define thermal diffusivity and time constant of a thermocouple and give an equation for each.
- Calculate the response time for a given thermocouple.
- Calculate and plot the temperature indicated by a thermocouple that is subjected to a sudden change of temperature, given necessary information about the thermocouple's properties.
- Compare and contrast the importance of mass, area, and bonding to material as these factors affect the relative response time of conventional and thin-film thermocouples.
- Given the appropriate equipment, measure the response time of a thin-film thermocouple and of a conventional thermocouple; and measure thermal diffusivity with a thermocouple.

## DISCUSSION

Heat flow is dependent on thermal conductivity (k) and specific heat (c) of the substance. In fact, the heat flow or heating rate of a substance is inversely proportional to the specific heat per unit volume, which is equal to $\rho c$, where $\rho$ is the material density. An important factor for determining heat flow is $k/\rho c$. This factor has the dimensions of $cm^2/s$, characteristic of a diffusion coefficient, and, therefore, has been given the descriptive term "thermal diffusivity" (to identify it as the diffusion coefficient of heat).

For changing or transient conditions of heat flow, the term $k/\rho c$ is important. The factor is significant because it determines how rapidly a material will accept and conduct thermal energy. Values of thermal diffusivity for a few metals are given in Table 1.

TABLE 1: THERMAL DIFFUSIVITY OF COMMON METALS

| Metal | Thermal Diffusivity ($cm^2/s$) |
|---|---|
| Aluminum | 0.86 |
| Brass | 0.34 |
| Copper | 1.14 |
| Nickel | 0.155 |
| Steel (carbon) | 0.121 |
| Steel (stainless) | 0.040 |

When a certain amount of heat energy is applied to a material, the heat energy moves (propagates) through the material in a finite time. In many practical cases, propagation can be defined in terms of a time constant. The changing temperature T of the substance being heated as a function of time t follows a relation analogous to Equation 1.

$$T = T_f - (T_f - T_i)\, e^{-t/t_c} \qquad (1)$$

where

$T_f$ = final temperature.

$T_i$ = initial temperature.

$t_c$ = time constant of the particular substance.

When $t = 0$, $e^{-t/t_c} = 1$, and $T = T_i$. When t becomes very large, $e^{-t/t_c} \approx 0$, and $T = T_f$.

Equation 1 supposes that heat energy is delivered beginning at time $t = 0$ to a material at temperature $T_i$. When temperature is being measured at a distance from the position at which heat energy is being supplied to a material with thermal diffusivity k, the thermal time constant often is given approximately by Equation 2.

$$t_c = d^2/k \qquad (2)$$

where

d = distance from point of temperature measurement to point at which heat energy is applied.

k = thermal diffusivity.

This approximate relation is not applicable in all cases; still, it defines a measure of the thermal time constant.

Obviously, all the above concepts hold equally well whether the substance is heated or cooled; thus, heat flow into or away from a given substance (and use the concept of a time constant) have equal validity.

Example A illustrates the use of Equations 1 and 2 in studying temperature versus time for a heated aluminum bar.

**EXAMPLE A: GRAPHICAL SOLUTION OF BAR-END TEMPERATURE**

Given: An aluminum bar 10 cm long is initially at temperature 20°C. At time $t = 0$, heat is supplied to one end; heating continues until the bar reaches a uniform steady state temperature of 80°C.

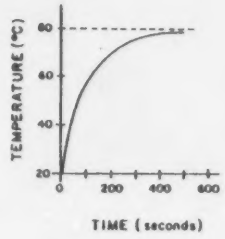
Find: Use Equations 1 and 2 to make a graph of temperature versus time for the far end of the bar.

Solution: From Equation 2 and Table 1:

$t_c = 10^2/0.86 = 116.3$ seconds

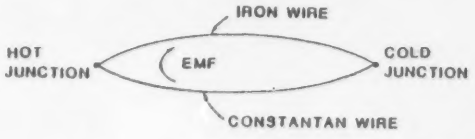
Substituting in Equation 1 gives:

$T = 80 - 60\, e^{-t/116.3}$

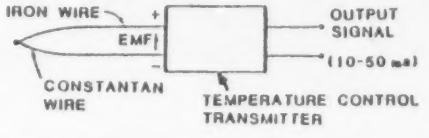



The above drawing gives results obtained by graphing this equation. An exponential approach to the final temperature of 80°C is observable. A thermocouple monitoring temperature of the end of the bar would show an output of a similar form.

Propagation of heat across an interface of two different materials is also important. A thermocouple (Figure 1) can be attached to a solid sample, or it can be immersed in a fluid. Heat must be transferred between the material (either solid or fluid) and the thermocouple. An exact description of heat transfer across such a boundary is very complicated. One need only note that heat transfer across a boundary involves a factor called "surface heat transfer coefficient" (h). This coefficient defines how much heat energy is transferred across a unit area of the boundary during a unit of time for a unit of temperature




(a) AN IRON-CONSTANTAN THERMOCOUPLE




(b) A TYPICAL THERMOCOUPLE AND TRANSMITTER

FIG. 1

difference. The surface coefficient (h) has units of calories/ $cm^2$-s-°C. For two solids, the surface coefficient will depend on how well the surfaces are in contact. For a solid and a liquid, the value depends on thickness of a boundary layer of liquid, across which temperatures vary. For instance, two solids separated by a thin layer of grease 0.1 mm thick will have a surface heat transfer coefficient of approximately 0.01 calories/ $cm^2$-s-°C.

Surface heat transfer coefficients are important in determining how heat is transferred from a solid to a thermocouple.

The actual, in-use response time of a thermocouple will be affected by two factors:

(a) Time taken for heat to be conducted from its point of application in the material to the position of the thermocouple in the material. (Note that this time lapse has nothing to do with the properties of the thermocouple.)

(b) Time taken for heat to be transferred from a material to the thermocouple and for the thermocouple to respond. (Note that this is what usually is referred to as the thermocouple response time.)

In many cases (a) above may be estimated by using concepts described in relation to Equations 1 and 2, and Example A. Often this time will be very short. For example, if a thermocouple is plunged suddenly into a liquid of a different temperature, the time represented by factor (a) will be negligible, and response of a thermocouple to an instantaneous change of temperature is governed essentially by factor (b).

For an instantaneous change in temperature from $T_1$ to $T_2$, temperature T indicated by the thermocouple will be given by Equation 3.

$$(T - T_2) = (T_1 - T_2)\, e^{-t/t_c} \qquad (3)$$

Thus, response will be governed by the time constant $t_c$.

**EXAMPLE B: CALCULATION OF TEMPERATURE WITH VARYING TIME**

Given: A thermocouple at temperature 20°C is plunged into a bath of boiling water. The thermocouple has a response time of 0.1 second.

Find: Temperature the thermocouple indicates after 0.1, 0.2 and 1.0 seconds.

Solution: Substitute into Equation 3 to obtain the temperature T as a function of time t.

$t_c = 0.1$ s

$T_1 = 20°C$ (initial temperature)

$T_2 = 100°C$ (boiling point of water)

$T = 100 - 80\, e^{-t/0.1}$

When $t = 0.1$ s, $T = 100 - 80\, e^{-1} = 70.6C°$

When $t = 0.2$ s, $T = 100 - 80\, e^{-2} = 89.2C°$

When $t = 1$ s, $T = 100 - 80\, e^{-10} = 99.996C°$

The thermal time constant $t_c$ of a thermocouple is given by Equation 4.

$$t_c = \frac{mc}{k_h A} \qquad (4)$$

where

$m$ = mass of thermocouple.
$c$ = specific heat of thermocouple.
$k_h$ = coefficient of surface heat transfer from material to thermocouple.
$A$ = area of contact between thermocouple and material.

Example C illustrates the use of Equation 4 in calculating the time constant for a thermocouple of given properties and given surface heat transfer coefficient ($k_h$).

**EXAMPLE C: CALCULATION OF THERMOCOUPLE TIME CONSTANT**

Given: A thermocouple of mass 0.002 g and specific heat 0.08 cal/g - c° is attached to a sample through a layer of grease 0.1 mm thick. The area of contact between thermocouple and sample is 0.02 $cm^2$.

Find: Time constant of the thermocouple.

Solution: Use Equation 4, with the value for h of 0.01 cal/$cm^2$ - s - °C. Substituting in Equation 4 produced this result:

$$t_c = \frac{mc}{k_h A}$$

$$t_c = \frac{(0.002)(0.08)}{(0.02)(0.01)}$$

$$t_c = 0.8 \text{ second}$$

Usually, time constants of the thermocouple should be small, in order that the thermocouple can monitor changing temperature without a large time delay. Equation 4 indicates four important factors for reducing the time constant:

- Mass of a thermocouple should be as small as possible.
- The thermocouple should be made of material with low specific heat, if possible. Often this will not be possible, because choice of materials will be governed by thermoelectric response. Materials will be chosen to give a large voltage change for a given temperature change, rather than for low values of specific heat.
- Area of contact between thermocouple and sample should be as large as possible.
- Thermocouple should be in good thermal contact with sample; this will lead to a large value of surface heat transfer coefficient ($k_h$).

In some cases, a compromise between response time and ruggedness of the thermocouple must be made. If a thermocouple is to be used in an environment in which vibration and mechanical shock are present, the necessity may arise to increase its mass and modify the attachment to make installation more durable.

These changes can decrease the response time; thus, a low-mass device for rapid response must be balanced against a larger device for improved tolerance to mechanical shock and vibration.

Efforts to attain a fast thermocouple response center on the use of thin metallic films, evaporated on the sample. Reference to Equation 4 reveals why thin-film thermocouples can offer faster responses.

A conventional thermocouple often is made of two wires, bonded together, with a small drop of melted and refrozen metal. Thermocouples are attached to a sample with some good heat-conducting material, such as certain greases or adhesives, to make them stick. The drop of metal sets a lower limit to the mass. It also determines the area of contact between thermocouple and sample. A bonding material also limits thermal conduction from sample to thermocouple. According to Equation 4, all these factors influence the time constant. Again, use of thin metal film improves response time.

In a thin-film thermocouple, metals are evaporated to form layers between $10^{-5}$ and $10^{-4}$ cm thick, reducing the mass to a minimum. They are deposited directly on the material; thus, there is no need for an intermediate material to attach the thermocouple. Thermal contact with the sample is maximized, therefore, and surface heat transfer coefficient is high. A thin film also may be spread over a larger surface area than the dimension of a conventional thermocouple.

In thin-film thermocouples, time constants as low as one microsecond have been achieved. Thus, a thin-film thermocouple can offer truly rapid response in situations of changing temperature. In contrast, conventional thermocouples (two thin wires with a small bead of metal at the junction) may have response speeds of a few tenths of a second — five orders of magnitude larger!

Investigations of methods of producing conventional thermocouple junctions have been carried out (reference Green and Hunt). Thin wires around 0.01-inch diameter were used. Wires were chromel (nickel with 10% chromium) and alumel (nickel with 3% manganese, 2% aluminum and 1% silicon). Wires were spot welded with a resistance welder under carefully-controlled conditions to form a bead no larger than the original wire diameter. Even with these precautions, response speeds were found to be about 10 milliseconds.

Workers using evaporated thin films of copper and constantan (copper with between 35% and 50% nickel) were able to obtain response times of less than one microsecond. The thermocouples were formed by the evaporation of metals in a vacuum and by allowing the metal vapor to condense on glass substrates. Thickness of the layers was about $1.2 \times 10^{-5}$ cm. Response time was measured by heating the junctions with a pulsed laser (reference Chopra, Bahl and Randlett).

More recently, fast thermocouples have been formed by a thin tube of metal forced over a fine wire, with 0.0005-in thick insulation between elements. Contact between tube and wire is formed by the evaporation of a thin film of metal on the end of the structure. This variation of commercially-available thin-film thermocouples also has response times as small as one microsecond.

Use of thin-film thermocouples involves extra expense and labor, compared to conventional thermocouples. In many applications, increased speed of response is not needed, and conventional thermocouples are satisfactory. For example, in monitoring a furnace, the temperature will not change significantly on a microsecond time scale. Conventional thermocouples with time-constant ranges from tenths of a second, or perhaps seconds, will be fast enough to give an accurate measure of changing temperature.

Thermocouples are routinely used in industry to monitor furnace or over temperatures under conditions of heating or cooling. Thermocouples are an important element in a feedback system that stabilizes temperature. In this application, the thermocouple must respond rapidly to varying temperature.

Thermocouples are employed to measure temperature of flowing fluids under conditions of variable temperature. Once again, if temperature of the fluid changes rapidly, the thermocouple must sense the change rapidly.

Thermocouples are used in nuclear reactors, where speed can be critical. As conditions in a reactor change, the reactor core may heat up. To keep temperature within acceptable limits, sensors must measure the temperature change rapidly, in order that responsive control systems can reduce heating.

Fast-response thermocouples are used in a variety of specialized scientific research and engineering applications, such as the measurement of temperatures inside gun barrels and the recording of heating effects produced by high-power pulsed lasers. They are also used in measuring heatup of rocket nose cones.

## SUMMARY

The response time of thermocouples is unimportant in many temperature-measurement applications; however, in other applications — such as nuclear reactors, temperature measurement of swift fluid flow, and aerospace equipment — a very fast response is required. The thin-film thermocouple provides the faster response time of the two major types (the thin-film and the wire

types). Response time depends upon the following characteristics of a thermocouple: (1) its size, (2) its material, and (3) its bonding to the material under study.

# RESPONSE TIME OF CONVENTIONAL AND OF THIN-FILM THERMOCOUPLES

## MATERIALS

Dewar flask of liquid nitrogen (or a beaker of water and ice if liquid nitrogen is not available).
Wires for a thermocouple (e.g., copper and constantan, or other metals if available).
Soldering iron and solder.
Oscilloscope (should have vertical sensitivity of a few millivolts/division).
Thin-film thermocouple. (Medtherm Corp., P. O. Box 412, Huntsville, Alabama 35804, or equivalent).
Electrical leads and wires as needed.
Oscilloscope camera and Polaroid film, if available.

## PROCEDURE

In this experiment, the student will measure the response time of thermocouples by plunging them in a liquid at a different temperature and observing a voltage pulse. Results for a conventional thermocouple are to be compared with a thin-film thermocouple.

1. Fill a small dewar flask with liquid nitrogen. (If not available, use a beaker of water and ice.) Liquid nitrogen, if available, will give easier-to-measure, larger changes of voltage.

2. Prepare a thermocouple (copper and constantan or other appropriate metals). Twist ends of wires together; solder them. Make solder bead as small as possible. Attach opposite ends to oscilloscope input.

3. Plunge thermocouple into cold liquid. The sudden change of temperature will generate a change in voltage, which can be viewed on oscilloscope. Several trials may be required to adjust sweep speed and vertical sensitivity. After each trial, allow thermocouple to warm to room temperature. Oscilloscope trace should appear something like drawing in Figure 2. Total change in voltage will

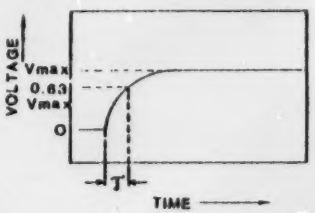



FIG. 2

be a few millivolts if liquid nitrogen is used; less if water and ice are used.

4. On oscilloscope trace, measure time at which voltage change has reached 63% of its final value. Simplify this step with a scope camera and Polaroid film, if available. Photograph scope trace and make measurements on photograph. (Remember from discussion in Module 12-0, page 6, that a parameter has reached 63% of its final value when time elapsed is equal to time constant. This measurement thus gives the constant directly. The 63% level is indicated at right of Figure 2.)

5. Repeat steps 3 and 4 with a thin-film thermocouple. Compare time constants obtained with two types of thermocouples.

6. Why does the thin-film thermocouple have a faster response?

## THERMAL DIFFUSIVITY OF A METAL

### MATERIALS

Dewar flask of liquid nitrogen (or a beaker of water and ice if liquid nitrogen is not available).
Wires for a thermocouple (e.g., copper and constantan, or other metals if available); same or similar type as used in first experiment.
Strip chart recorder. (Suppliers include Honeywell, Inc., Test Instruments Division, Denver, Colorado. Honeywell, model Electronik 19 has a 0.1-millivolt scale and would be suitable. Esterline-Angus, Indianapolis, Indiana, is another possible supplier.).
Stirring rod.
Adhesive (e.g., General Radio cement).
Stainless steel rod about 10 cm long (rods of other metals are acceptable).
Clamp and stand to support rod.
Ruler.

### PROCEDURE

In this experiment, the student will use a thermocouple under conditions of changing temperature to determine thermal diffusivity of a metal.

1. Attach thermocouple to one end of metal rod, using adhesive.

2. Attach thermocouple leads to strip chart recorder.

3. Turn on recorder; place other end of rod into liquid nitrogen (or beaker of water and ice). Apparatus is sketched in Figure 3. Support rod with a suitable clamp.

4. Observe change of voltage on strip chart. Stir liquid with stirrer in order that cold liquid is kept in contact with ends of rod. Metal will cool gradually. As far end of rod cools, voltage indicated by chart recorder will change. Under conditions described here, a few hundred seconds may be required for voltage to reach its final steady value. Use a slow chart speed.

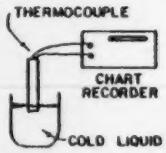



FIG. 3

5. From strip chart, determine time $t_c$ at which voltage has changed 63% from initial value to final value.

6. Analyze results, using the following equation for approximate voltage V as a function of time t:

$$V = V_f + (V_i - V_f)\, e^{-D\pi^2 t/4\ell^2}$$

where

$V_i$ = initial value of voltage.

$V_f$ = final value of voltage.

D = thermal diffusivity of metal.

$\ell$ = length of rod (measured with ruler).

This equation represents the approximate temperature of one end of a rod of material, when the other end is suddenly placed in contact with a well-stirred fluid at time zero. Note that at time zero, the exponential factor is one, and —

$$V = V_f + (V_i - V_f) = V_i.$$

At large values of time t, the exponential factor approaches zero, and —

$$V = V_f.$$

From this equation, the value of the time constant $t_c$ as determined from step 5 is —

$$t_c = 4\ell^2/D\pi^2.$$

Thermal diffusivity k is derived as follows:

$$D = 4L^2/\pi^2 t_c.$$

Use measured values to calculate k. Compare the results to values in Table 1, and explain any difference in values.

## SELF CHECK

1. Define thermal diffusivity, using an equation. Also using an equation, define time constant of a thermocouple.

2. A thermocouple has a mass of 0.006 g, a specific heat of 0.15 cal/g degree, a contact area of 0.01 $cm^2$ and a surface heat transfer coefficient of 0.03 cal/$cm^2$-s-°C. What is time constant of this thermocouple?

3. The thermocouple of problem 2 is at 20°C. It is suddenly plunged into liquid nitrogen (temperature -196°C). Graphically plot temperature indicated by the thermocouple as a function of time. Calculate indicated temperature at several values of time, indicating initial rapid change and approach to a steady final temperature.

4. Compare and contrast mass, area, and bonding to material as these factors affect relative response time of conventional and thin-film thermocouples.

## REFERENCES

American Society for Testing and Materials. Manual on the Use of Thermocouples in Temperature Measurement. ASTM Special Technical Publication 470. Philadelphia, PA: ASTM, 1970. See especially Section 9.

Chopra, K. L.; Bahl, S. K.; and Randlett, M. R. "Thermopower in Thin-Film Copper Constantan Couples." Journal of Applied Physics, 39 (1968): 1525-1528.

Green, S. J. and Hunt, T. W. "Accuracy and Response of Thermocouples for Surface and Fluid Temperature Measurements." Temperature, Its Measurement and Control in Science and Industry. Vol. 3, Part 2, p. 695. New York: Reinhold, 1962.

Jakob, M. Heat Transfer. Vol. 2. New York: John Wiley and Son, 1957.
See especially Chapter 33.

Kinzie, P. A. Thermocouple Temperature Measurement. New York: John Wiley and Son, 1973.
See especially Chapter 1.

Kovacs, A. and Mesler, R. B. "Making and Testing Small Surface Thermocouples for Fast Response." Review of Scientific Instruments. 35 (1964); 485-488.

Medtherm Corporation. "Measure Surface Temperature with a Response Time as Little as 1 Microsecond." Bulletin 500. Huntsville, AL: Medtherm Corporation, 1974.

Murdock, J. W.; Foltz, C. J. and Gregory, C. "A Practical Method of Determining Response Time of Thermometers in Liquid Baths." Journal of Engineering for Power (Transactions of the ASME), p. 27, January 1963.

UTC

UNIFIED TECHNICAL CONCEPTS

MODULE 12 T 2

# TIME CONSTANTS OF HEATING AND COOLING

## THERMAL APPLICATION OF TIME CONSTANTS

Technical Education Research Center – Southwest

TERC – SW

## INTRODUCTION

Control of the rate of heating and cooling is an important function in many industrial situations. For example, the cooling rate of hot glass for telescope mirror blanks must be controlled during the manufacturing process. If the blanks were cooled at too fast a rate, the outside layers of glass would be cool, while the layers underneath would remain hot. The resulting thermal stress would cause cracks to occur, damaging thousands of dollars worth of glass work. Time constants associated with the response of temperature-measuring devices, such as liquid-filled thermometers, are important factors to consider in the design of an accurate temperature-measuring system. This module describes time constants for heating and cooling and relates the time constants to the rate of temperature changes of a body as it heats or cools. In the laboratory, the student will determine how different surface characteristics of a hot body affect the time constant and the rate at which it cools.

## PREREQUISITES

Student should have completed Concept Module 12-0, "Time Constants."

## OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain how time constants are used to measure the rate at which temperature approaches an equilibrium state.
- Given an initial temperature, ambient temperature, and time constant for a material body, determine its temperature at any time.
- Given the appropriate equipment, calculate time constants of materials with different thermal resistances.

# DISCUSSION

When heat flows into a body, and no mechanical work is done by the body, all the added energy shows up as stored internal energy, as evidenced by a rise in temperature of the body. A release of this stored energy to its surroundings is evidenced by a decrease in the body temperature. The direction of heat flow is governed by the law that states, "Heat travels naturally from a hot body (at high temperature) to a cooler body (at low temperature)." When a body is at the same temperature as its surroundings, no heat is transferred; and the body is said to be in "equilibrium" with its surroundings. The larger the temperature difference between two different bodies, the faster the heat flow between them.

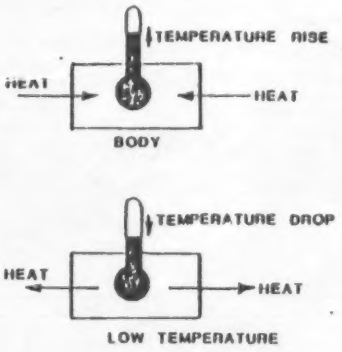



FIG. 1

Heat is transferred by three methods — conduction, convection, and radiation. Conduction is a method of heat transfer that occurs on a molecular scale, and is the predominant method of heat transfer through solids. Convection is a method of heat transfer that occurs by movement of hot and cold masses of material. Convection is an important method of heat transfer in fluids. Radiation is a method of heat transfer in which heat is transferred by means of electromagnetic radiation. The rate at which heat is transferred by this method is strongly dependent upon the surface characteristics of the body. Black-colored objects radiate heat at the fastest rate. Automotive racers use car engines of increasing displacement, causing problems of excessive heat. Blackened engine blocks, rocker arm covers, and oil pans aid in getting the heat out of the automobile engine at the fastest rate. Car radiators are painted black for the same reason. Silver, chrome, or mirror-surfaced bodies radiate heat at a slower rate. A thermos bottle uses this characteristic to reduce the rate at which its contents loses or gains heat. Most often these methods of heat transfer occur together, so that total heat transfer occurs as a result of a combination of conduction, convection, and radiation. An example of such a combined effect is shown in Figure 2.

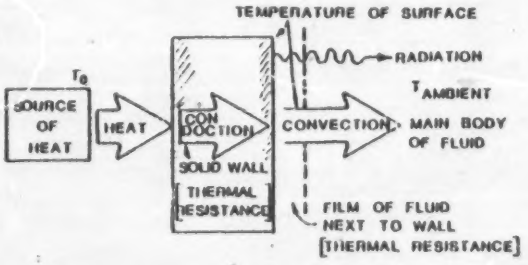
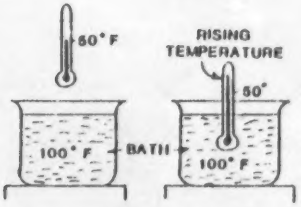



FIG. 2

Often, in practical applications, the total rate of heat flow is difficult to describe; however, the total rate of heat flow is dependent upon the difference in temperature between the source of heat and a surrounding fluid, which maintains a constant ambient temperature (Figure 2). The total heat flow rate also is affected by the total combined thermal resistance of conduction, convection, and radiation effects.

## THE RATE OF HEATING

The process of heating a cool body to a higher temperature occurs in our daily lives frequently. The doctor inserts a thermometer (cool body) in the mouth of a patient (hot surroundings) and waits three or four minutes while the thermometer comes to the temperature of the patient's body. A housewife places a turkey (cool body) in the oven (hot surroundings) and then waits for the meat to reach cooking temperature. In each case, the hot surroundings maintain a constant temperature while the cooler body gradually increases its temperature.



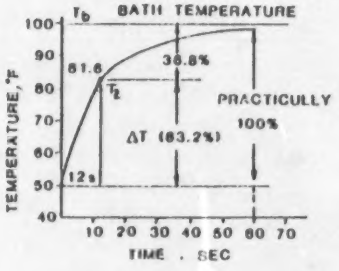




FIG. 3

Consider a thermometer initially at $T_o$ (50°F) being plunged into a bath at temperature $T_B$ (100°F). The temperature is recorded at intervals of 5 seconds, as shown in Figure 3. The temperature of the thermometer at any time is given by Equation 1.

$$T = T_o + (T_B - T_o)[1 - e^{-t/\tau}] \qquad (1)$$

where

$T$ = temperature of thermometer at time $t$.
$T_o$ = temperature of thermometer at time $t = 0$.
$T_B$ = constant temperature of hot surroundings (bath temperature).
$e$ = exponential base = 2.71828.
$\tau$ = time constant of thermometer.

As discussed in Concept Module 12-0, the time constant (naperian base) is the time required for a body to reach 63.2% of the difference $(T_B - T_o)$. For the example shown in Figure 3, the time constant of the thermometer is 12 seconds. After the 12 seconds, the temperature has risen to 81.6°F. After 5 time constants (5 x 12 seconds = 60 seconds), the thermometer, for all practical purposes, has risen to 100°F. The time constants associated with half-life processes are not normally used in thermal work.

The time constant also can be defined as the ratio of the temperature difference $(T_B - T)$ and the rate of temperature change at that temperature. It is given by the relation indicated in Equation 2.

$$\tau = \frac{(T_B - T)}{(\frac{\Delta T}{\Delta t})} \qquad (2)$$

where

$T_B$ = temperature of hot surroundings.
$T$ = temperature at time $t$.
$(\Delta T/\Delta t)$ = rate of temperature change at $T$ (slope of tangent line at point $t$, $T$ on graph).

Equation 2 can be rearranged to solve for rate of temperature change, as indicated by Equation 3.

$$\left(\frac{\Delta T}{\Delta t}\right) = \frac{T_B - T}{\tau} \qquad (3)$$

Equations 1, 2, and 3 can be applied to any thermal body placed in hotter surroundings of constant temperature. These equations are used to solve a problem in Example A.

**EXAMPLE A: HEATING OF A BODY TO THE CONSTANT TEMPERATURE OF ITS SURROUNDINGS**

**Given:** A thermometer initially at 50°F is plunged into a hot-water bath of 200°F. The system time constant is 12 seconds.

**Find:**
(a) The temperature after one time constant.
(b) The time required for the thermometer to be practically the same temperature as the bath.
(c) The initial rate of temperature change at 50°F.

**Solution:**

(a) $T = T_o + (T_B - T_o)[1 - e^{-t/\tau}]$

$= (50°F) + (200°F - 50°F)\left[1 - e^{-\left(\frac{12\ s}{12\ s}\right)}\right]$

$= 50°F + (150°F)[1 - e^{-1}]$

$T = 144.8°F$

(b) Time for thermal equilibrium = 5 x time constant

$= (5)(12\ s)$

$= 60\ s$

(c) Solve for $(\Delta T/\Delta t)$, using Equation 3:

$$\left(\frac{\Delta T}{\Delta t}\right) = \frac{T_B - T}{\tau}$$

$$= \frac{(200°F - 50°F)}{12\ s}$$

$$\left(\frac{\Delta T}{\Delta t}\right) = 12.5\ \frac{F°}{s}$$

Regardless of the temperature of the bath, the thermometer will achieve 63% of the difference between the initial temperature of the thermometer and the temperature of bath I one time constant. After 5 time constants, the thermometer almost will achieve equilibrium, regardless of the temperature of the hot surroundings. Of course, the value of the time constant itself depends upon the bath temperature.

## THE RATE OF COOLING OF A HOT BODY

The process of cooling a hot body in surroundings of constant cooler temperature occurs in everyday circumstances, such as the heat-treating of metal parts (quenching), the freezing of foods, and the response of temperature-sensing instruments.

Consider a thermometer which is initially in a hot water bath of 170°F and then is withdrawn to a still air at 70°F. The decreasing temperature of the thermometer is recorded every 50 seconds, as illustrated in Figure 4. In such a case, the temperature is given by Equation 4.

$$T = T_{AMB} + [T_o - T_{AMB}]e^{-t/\tau} \qquad (4)$$

where

$T_{AMB}$ = ambient temperature.

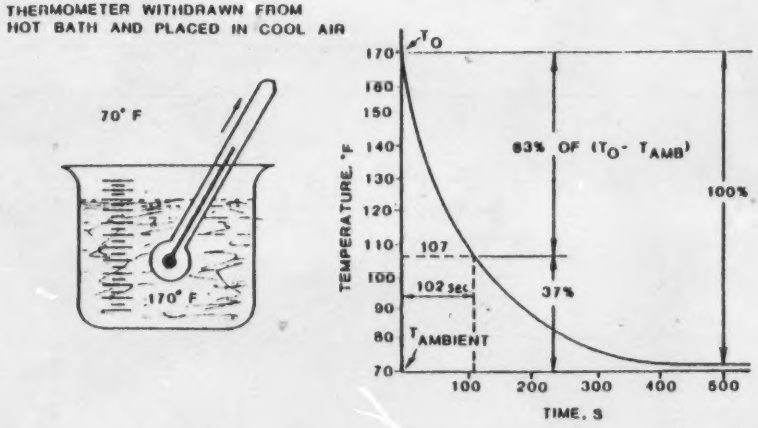



FIG. 4

The time constant will be the time required for the temperature of the body to cool to 63% of $(T_O - T_{AMB})$. The time constant also is related to the rate of temperature change by Equation 5.

$$\tau = \frac{(T - T_{AMB})}{(\frac{\Delta T}{\Delta t})} \qquad (5)$$

The time required for the hot body to reach ambient temperature is again equal to 5 time constants. In the example in Figure 4, the time constant is 102 seconds. The temperature drops to 107°F after one time constant. After 500 seconds the thermometer essentially reaches the ambient temperature of 70°F.

Equations 4 and 5 are employed to solve a sample problem in Example B.

**EXAMPLE B: COOLING OF A BODY TO THE CONSTANT TEMPERATURE OF ITS SURROUNDINGS**

Given: A thermometer is immersed in a 120°F hot water bath and is withdrawn and left in still air at 70°F. The thermometer has a time constant of 102 seconds.

Find:
(a) Temperature after one time constant.
(b) Time required for the thermometer to be practically at still-air temperature.
(c) Initial rate of temperature change at 120°F.

Solution:

(a) $T = T_{AMB} + [T_O - T_{AMB}]e^{-t/\tau}$

$= (70°F) + [120°F - 70°F]e^{-(\frac{\cancel{102\ s}}{\cancel{102\ s}})}$

$= 70°F + (50°F)\ e^{-1}$

$T = 88.4°F$

(b) $T_{equilibrium} = 5\ \tau$

$= 5\ (102\ s)$

$T_{equilibrium} = 510\ s$

(c) $\frac{\Delta T}{\Delta t} = \frac{T - T_{AMB}}{\tau}$

$= \frac{(120°F - 70°F)}{102\ s}$

$\frac{\Delta T}{\Delta t} = 0.49\ \frac{F°}{s}$

Notice that no matter what the initial temperature of the body is, the body essentially will reach equilibrium in 5 time constants.

## FACTORS AFFECTING THE TIME CONSTANTS FOR HEATING AND COOLING BODIES

The value of the time constant for a given body depends upon many factors. The thermal resistance of the material of the body, the specific heat of the material, the surface area of the body exposed to the surroundings, and the mass of the body all affect the rate at which heat passes into, or out of, a body. Another factor that affects the rate of temperature change of a body is the state of motion of the fluid surroundings; for example, a thermometer in still air has a larger time constant than one in moving air.

## SUMMARY

The transient processes of heating and cooling a body in a constant-temperature, ambient environment are described by use of an important factor called a "time constant." Specific equations involving temperature, time, and the time constant aid in the modeling of the thermal behavior of a heating or cooling body.

## THERMAL TIME CONSTANTS

### MATERIALS

3 brake fluid cans of same size and materials (two painted mirror-finish chrome or silver, one painted flat black).
3 rubber stoppers to fit opening in cans (stoppers bored for thermometer).
1 asbestos or cork sheet 1/4" thick as insulating jacket to one of silvered cans.
3 thermometers (0 to 100°C, 1C°).
1 small bottle glycerin (thermometer lubricant).
3 beakers (500 ml volume).
1 beaker tongs.
Wristwatch or clock.
1 room thermometer.

### PROCEDURE

1. Arrange brake fluid cans as illustrated in Figure 5. Make sure that they are separated from each other by about 50 cm and that they are supported on insulating paper towels.

2. Heat a large quantity of water to boiling temperature or, at least, above 95°C.

3. While water is heating, check arrangement in Figure 5 to ensure that the thermometers are adjusted about 2 centimeters above bottoms of cans.

4. When water has achieved its high temperature, fill each can with an equal volume (300-400 ml) of hot water. Insert the thermometers and rubber stoppers as shown in Figure 5. Notice the rise in temperature of the thermometer.

5. Read the room thermometer, and record this ambient temperature in the Data Table.

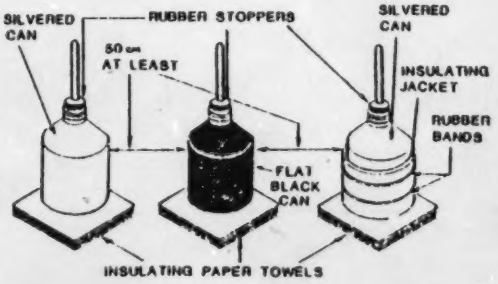



FIG. 5

6. After thermometers have reached their maximum temperature (in about 3 or 4 minutes), begin recording temperature of each can every two minutes for the first 10 minutes and then once every 10 minutes for 2 1/2 hours. Record time and temperature in Data Table.

7. Plot temperature vs. time curve for each can on the same graph (i.e., three curves on same graph). Identify each curve on graph with proper can.

8. Does the black can cool at a faster rate than the silvered can? If so, why?

9. Which can, the silvered or insulated silvered, cools at a faster rate? Why?

10. From the graph, which can has the largest time constant? The smallest time constant?

11. Determine the time constant for each can from the curve.

12. Based upon the time constants, compute the time required for the cans to achieve room temperature.

| DATA TABLE | | | |
|---|---|---|---|
| TIME (MIN) | TEMPERATURE OF SILVERED CAN (°C) | TEMPERATURE OF BLACK CAN (°C) | TEMPERATURE OF INSULATED CAN (°C) |
| 0 | | | |
| 2 | | | |
| 4 | | | |
| 6 | | | |
| 8 | | | |
| 10 | | | |
| 20 | | | |
| 30 | | | |
| 40 | | | |
| 50 | | | |
| 60 | | | |
| 70 | | | |
| 80 | | | |
| 90 | | | |

| | | | |
|---|---|---|---|
| 100 | | | |
| 110 | | | |
| 120 | | | |
| 130 | | | |
| 140 | | | |
| 150 | | | |
| ROOM TEMPERATURE (AMBIENT) = _____ °C | | | |

## SELF CHECK

1. Thermometer A has a time constant of 8 seconds. Thermometer B has a time constant of 6 seconds.

   (a) Which thermometer reaches equilibrium quicker?
   (b) For the same initial temperature and hot environment temperature, which thermometer has the larger initial rate of temperature change?

2. Assuming that 4 minutes is about the time required for a doctor to get an accurate temperature measurement of a patient (equilibrium of thermometer and body temperature), what is the time constant of the thermometer used?

3. A chrome-constantan thermocouple has a time constant of 0.7 second, and is initially at a temperature of 80°F. It is plunged into a hot oil bath of 300°F. What is the temperature of the thermocouple after 0.5 second?

4. A 200-lb aluminum engine block cools from 150°F to 100°F in 15 min. Assume that the temperature drops exponentially with time. Surrounding air is 70°F. What is the time constant of the block? What is the temperature after 20 minutes?

5. A small thermocouple cools from 100°C to room temperature in about 35 seconds. If the thermocouple is heated in a flame to 500°C, what is its temperature 14 seconds after the flame is removed? Room temperature is 20°C.

6. A body cools at a rate of 0.25 C°/s when its temperature is 20°C above the ambient temperature. What is the cooling rate when the temperature is 60°C above its surroundings?

# REFERENCES


Benedict, Robert P. Fundamentals of Temperature, Pressure, and Flow Measurements. 2nd ed. New York: John Wiley and Sons, 1977.

Dierauf, Edward J., Jr., and Court, James E. Unified Concepts in Applied Physics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1979.

Doebelin, Ernest O. System Dynamics: Modeling and Response.

Fribance, Austin E. Industrial Instrumentation Fundamentals. New York: McGraw-Hill Book Company, Inc., 1962.